Dynamics of Machines ME 6505

Unit I

Force Analysis

Dynamic force analysis

When the inertia effect due to the rotating / reciprocating parts is considered in addition to the externally applied loads, such a force Analysis is known as dynamic force Analysis.

dynamic force Analysis are very important in operating speeds are high in m/c.

ex

Ic engines rotate @ very high speed, even the slightest eccentricity at the centre of mass from the axis of rotation causes very high magnitude of dynamic forces.

This may lead to vibrations, wear, noise (or) even m/c failure

Inertia:

The property of matter offering resistance to any change of its state of rest (or) of uniform motion in a straight line. is known as inertia ex human in bus

Inertia force:

The inertia of the body opposes (against) this external force applied & it is known as inertia force.

$\boxed{F = ma}$ (or)

m = mass of the body
a = acceleration of the body

inertia force is a imaginary force which when acts upon a rigid body, brings it in an equilibrium. It is numerically equal to accelerating force in magnitude but opposite in direction

Inertia force = + external force (accelerating) $F = +ma$

Inertia Torque

inertia of the body opposes this external torque applied & it is known as inertia torque.

inertia torque is an imaginary torque, which when applied upon the rigid body, Brings it in equilibrium position. it is equal to the accelerating torque in magnitude but opposite in direction

$$\boxed{IT = -\text{externally applied torque} = +I\alpha}$$

D'Alembert's principle

States that, the inertia forces & torques, & the external forces & torques, acting on a body together result in statical equilibrium.

$$\boxed{\Sigma F = 0 \ \& \ \Sigma M = 0}$$

The vector sum of all external forces & Inertia forces acting upon a system of rigid bodies is zero.

The vector sum of all external moments & Iner torque acting upon a system of rigid bodies is also zero.

App
used to reduce a dynamic analysis problem in to an equivalent problem of static equilibrium.

condition ~~of~~ for static & dynamic equilibrium?

static eq

1. The vector sum of all forces acting on a body should be zero ($\Sigma F = 0$).
2. The vector sum of all moments about any arbitrary point is zero. ($\Sigma M = 0$)

condition of for dynamic equilibrium are the same, except for the ~~th~~ fact that the inertia forces & couples are included in the respective equations.

Dynamic Analysis of Reciprocating steam Engines

velocity & acce of the reciprocating parts in engine

The velocity & acceleration of various parts of reciprocating mechanism can be determined, both analytically & graphically.

methods

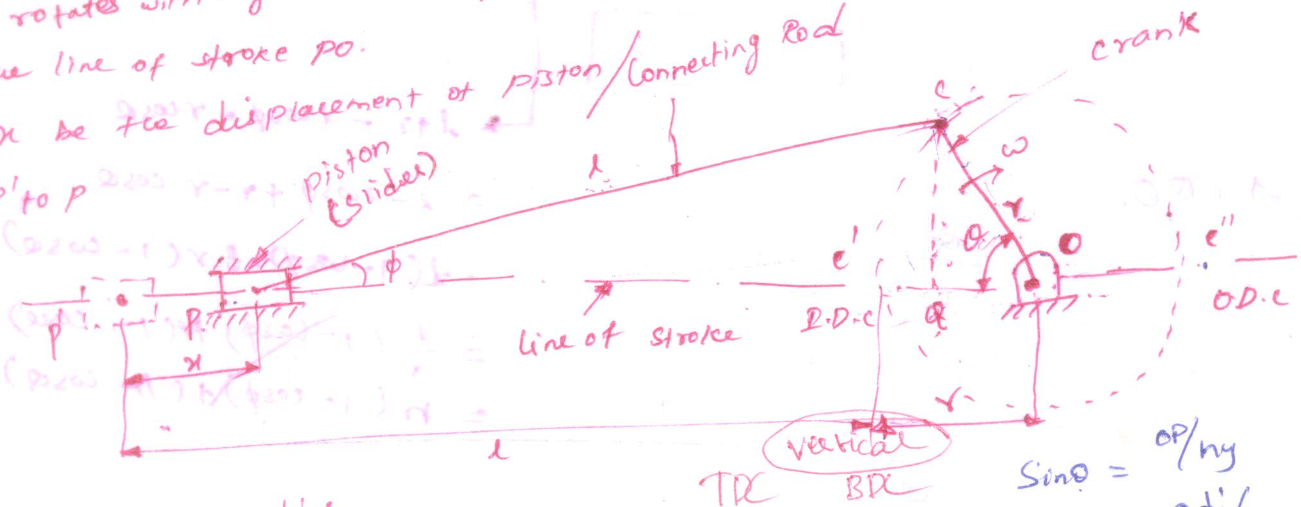
1. Analytical & graphical methods.

- Klein's construction
- Ritterhau's "
- Bennett's "

reciprocating steam engine mechanism OCP shown.

crank "OC" rotates with angular velocity ω r/sec. & connecting rod PC makes angle ϕ with the line of stroke PO.

x be the displacement of piston from P' to P



$\sin\theta = op/hy$

$\cos\theta = adj/hy$

$\tan\theta = adj/op$

r - crank ~~angle~~ radius

ω - Angular velocity of crank rad/sec

l - Length of ~~crank~~ ~~angle~~ connecting Rod.

θ - Angle made crank with I.D.C

ϕ - Inclination of connecting rod to the line of storoke PO

$n = \frac{l}{r}$ = ration of length of connecting rod to radius of crank is known obliquity ratio

obliquity ratio

$\frac{\sin}{\cos} = \frac{op/hy}{adj/hy}$

Velocity of the piston (V_p)

displacement of the piston is given by

$$x = P'P = OP' - OP$$

$$= (P'c' + c'O) - (PQ + QO)$$

From $\triangle CPQ$, $PQ = l\cos\phi$

from $\triangle COQ$, $QO = r\cos\theta$

(2)

$$\therefore x = (l+r) - (l\cos\phi + r\cos\theta)$$
$$= r(1-\cos\theta) + l(1-\cos\phi)$$
$$= r\left[(1-\cos\theta) + \frac{l}{r}(1-\cos\phi)\right]$$

$n = \frac{l}{r}$ $\qquad \boxed{x = r[(1-\cos\theta) + n(1-\cos\phi)]}$ — (1)

$(PQ + QO)$

$$\cos\phi = \frac{\text{adj side}}{\text{hyp}} = \frac{PQ}{l}$$
$$\boxed{PQ = l\cos\phi}$$
$$\cos\theta = \frac{\text{adj side}}{\text{hyp}} = \frac{QO}{r}$$
$$\boxed{QO = r\cos\theta}$$

$$l + r - l\cos\phi - r\cos\theta$$
$$= l - l\cos\phi + r - r\cos\theta$$
$$= l(1-\cos\phi) + r(1-\cos\theta)$$
$$= \frac{l}{r}\cancel{(1-\cos\phi)} + (1-\cos\theta)$$
$$= n(1-\cos\phi) + (1-\cos\theta)$$

from Δ ~~CPQ~~, CQP

$$\sin\phi = \frac{op}{hy} = \frac{CQ}{l} \text{ (or) } \boxed{CQ = l\sin\phi}$$

from Δ CQO,

$$\sin\theta = \frac{op}{hy} = \frac{CQ}{r} \text{ (or) } \boxed{CQ = r\sin\theta}$$

thus,

$$l\sin\phi = r\sin\theta$$
$$\sin\phi = \frac{r\sin\theta}{l}$$
$$\boxed{\sin\phi = \frac{\sin\theta}{n}} \quad \text{— (2)}$$

$$n = \frac{l}{r}$$
$$\frac{1}{n} = \frac{r}{l}$$

$$\sin^2\phi + \cos^2\phi = 1$$
(or)
$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$\cos\theta = (1-\sin^2\theta)^{1/2}$$

wkt $\cos\phi = (1-\sin^2\phi)^{1/2}$

$$\cos\phi = \left[1 - \frac{\sin^2\theta}{n^2}\right]^{1/2} \quad \text{LCM}$$
$$\cos\phi = \left[\frac{n^2 - \sin^2\theta}{n^2}\right]^{1/2}$$
$$\cos\phi = \frac{(n^2-\sin^2\theta)^{1/2}}{(n^2)^{1/2}}$$
$$\cos\phi = \frac{(n^2-\sin^2\theta)^{1/2}}{n} \Rightarrow \frac{1}{n}(n^2-\sin^2\theta)^{1/2}$$

By expanding the above expression by binominal theorem, we ge

$$\cos\phi = 1 - \frac{1}{2} \times \frac{\sin^2\theta}{n^2} + \cdots \quad \text{[neglecting higher order terms]}$$

$$\boxed{1 - \cos\phi = \frac{\sin^2\theta}{2n^2}} \quad \text{— (3)}$$

$$x = r\left[(1-\cos\theta) + n\left(\frac{\sin^2\theta}{2n^2}\right)\right]$$
$$x = r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right]$$

Substituting equation ③ in equ ①, we get

$$x = r\left[(1-\cos\theta) + n(1-\cos\phi)\right] \quad \leftarrow \quad 1-\cos\phi = \frac{\sin^2\theta}{2n^2} \quad -③$$

$$x = r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right] \quad (i)$$

$a^2 \Rightarrow 2a$

$2\sin\theta$

$\cos\theta = -\sin\theta$

$\sin\theta = \cos\theta$

x^3

$3x^2$

Differentiating equation (i) with respect to θ, we get

$r(0 + (\sin\theta)) +$

$$\frac{dx}{d\theta} = r\left[\sin\theta + \frac{1}{2n} \times 2\sin\theta\cos\theta\right]$$

$$\frac{dx}{d\theta} = r\left[\sin\theta + \frac{\sin 2\theta}{2n}\right] \quad —④ \quad \leftarrow \sin 2\theta = 2\sin\theta\cos\theta$$

∴ velocity of the piston (or) velocity of P with respect to O,

$$V_{PO} = V_P = \frac{dx}{dt}$$

$$= \frac{dx}{d\theta} \times \frac{d\theta}{dt} \Rightarrow \frac{dx}{d\theta} \times \omega \quad \left[\because \frac{d\theta}{dt} = \omega\right]$$

velocity of piston

$$\boxed{V_{PO} = V_P = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]} \quad —(ii)$$

Acceleration of the piston (a_p): → change of Rate of velocity into angular velocity.

acceleration of the piston P is given by

$$a_p = \frac{dv_p}{dt} = \frac{dv_p}{d\theta} \times \frac{d\theta}{dt} \Rightarrow \frac{dv_p}{d\theta} \times \omega \quad . \quad \frac{d\theta}{dt} = \omega$$

differentiating eq ii with respect to θ, we get

$$(ii) \Rightarrow \frac{dv_p}{d\theta} = \omega \cdot r\left[\cos\theta + \frac{\cos 2\theta \times 2}{2n}\right]$$

$\sin\theta = \cos\theta$

$\sin 2\theta = \cos 2\theta$

$\frac{d}{d\theta} 2\theta = 2$

$$= \omega \cdot r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Substituting the value of $\frac{dv_p}{d\theta}$ in the above eq. we get,

acceleration of piston

$$\boxed{a_p = \omega^2 \cdot r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]}$$

Angular velocity of the connecting Rod (ω_{pc}) :

from the geometry of the fig, we find that

$$CQ = l \sin\phi = r \sin\theta$$

$$\sin\phi = \frac{r}{l} \sin\theta$$

$$\sin\phi = \frac{\sin\theta}{n} \qquad \left(\frac{r}{l} = \frac{1}{n}, \ \frac{l}{r} = n\right)$$

diff both sides with respect to time t, we get

$$\cos\phi \times \frac{d\phi}{dt} = \frac{\cos\theta}{n} \times \frac{d\theta}{dt} \qquad \left(\therefore \frac{d\theta}{dt} = \omega\right)$$

$$\frac{d\phi}{dt} = \frac{\cos\theta}{n} \times \frac{\omega}{\cos\phi}$$

Same as the angular velocity of point P with resp to C &

angular velocity of the connecting rod PC is equal to $\frac{d\phi}{dt}$ ∴

$$\omega_{pc} = \frac{d\phi}{dt} = \frac{\cos\theta}{n} \times \frac{\omega}{\cos\phi}$$

$$\boxed{\omega_{pc} = \frac{\omega}{n} \times \frac{\cos\theta}{\cos\phi}}$$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = (1 - \sin^2\theta)^{1/2}$

$\sin\phi = \frac{\sin\theta}{n}$

wkt

$$\cos\phi = (1 - \sin^2\phi)^{1/2}$$

$$= \left(1 - \frac{\sin^2\theta}{n^2}\right)^{1/2}$$

$$\therefore \omega_{pc} = \frac{\omega}{n} \times \frac{\cos\theta}{\cos\phi}$$

$$= \frac{\omega}{n} \times \frac{\cos\theta}{\left(1 - \frac{\sin^2\theta}{n^2}\right)^{1/2}}$$

$$= \frac{\omega}{n} \times \frac{\cos\theta}{\frac{1}{n}(n^2 - \sin^2\theta)^{1/2}}$$

$$\boxed{\omega_{pc} = \frac{\omega \cos\theta}{(n^2 - \sin^2\theta)^{1/2}}} \quad \text{—— (iii)}$$

Note

the $\frac{\sin^2\theta}{n^2}$ is small the

(or)

$$\omega_{pc} = \frac{\omega \cos\theta}{n}$$

$$= \frac{\omega \cos\theta}{(n^2 - 0)^{1/2}}$$

$$\omega_{pc} = \frac{\omega \cos\theta}{}$$

Angular Acceleration of the Connecting Rod (α_{pc})

wkt the angular acceleration of p with respect to c,

$$\alpha_{pc} = \frac{d(\omega_{pc})}{dt}$$

$$\frac{d(\omega_{pc})}{dt} = \frac{d(\omega_{pc})}{d\theta} \times \frac{d\theta}{dt} \qquad \frac{d\theta}{dt} = \omega$$

$$= \frac{d(\omega_{pc})}{d\theta} \times \omega$$

$x^3 = 3x^{3-1} = 3x^2$

$x^{1/2} = \frac{1}{2} x^{\frac{1}{2}-1}$

Diff eq (iii) with respect to θ, we get

$$\frac{d(\omega_{pc})}{d\theta} = \frac{d}{d\theta}\left[\frac{\omega\cos\theta}{(n^2-\sin^2\theta)^{1/2}}\right] \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2} \text{ (or) } \frac{vu' - uv'}{v^2}$$

$x^2 = 2x$

$\frac{d}{dx}(\cos\theta) = -\sin\theta$

$v^2 = (n^2-\sin^2\theta)^{1/2}$

$$= \omega\left[\frac{[(n^2-\sin^2\theta)^{1/2}(-\sin\theta)] - [(\cos\theta) \times \frac{1}{2}(n^2-\sin^2\theta)^{-1/2} \times (-2\sin\theta\cos\theta)]}{(n^2-\sin^2\theta)}\right]$$

$$= \omega\left[\frac{(n^2-\sin^2\theta)^{1/2}(-\sin\theta) + (n^2-\sin^2\theta)^{-1/2}\sin\theta\cos^2\theta}{(n^2-\sin^2\theta)}\right]$$

$$= -\omega\sin\theta\left[\frac{(n^2-\sin^2\theta)^{1/2} - (n^2-\sin^2\theta)^{-1/2}\cos^2\theta}{(n^2-\sin^2\theta)}\right]$$

dividing & multiplying by $(n^2-\sin^2\theta)^{1/2}$, we get

$\frac{1}{2} = \sqrt{\ }$ $\qquad -\frac{1}{2} = \frac{1}{\sqrt{\ }}$

$$= -\omega\sin\theta\left\{\frac{[(n^2-\sin^2\theta)^{1/2} - (n^2-\sin^2\theta)^{-1/2}\cos^2\theta]}{(n^2-\sin^2\theta)^{1=(b)}}\right\} \times \frac{(n^2-\sin^2\theta)^{1/2}}{(n^2-\sin^2\theta)^{1/2 = c}}$$

$= a^b \cdot a^c = a^{(b+c)} = a^{(1+\frac{1}{2})} = a^{\frac{2+1}{2}} = a^{3/2}$

$$= -\omega\sin\theta\left[\frac{\sqrt{(n^2-\sin^2\theta)} - \frac{\cos^2\theta}{\sqrt{(n^2-\sin^2\theta)}}}{(n^2-\sin^2\theta)^{3/2}}\right] \times (n^2-\sin^2\theta)^{1/2}$$

$-\frac{1}{n} \qquad \frac{n-1}{n}$

$$= -\omega\sin\theta\left[\frac{\sqrt{n^2-\sin^2\theta} - \cos^2\theta}{(n^2-\sin^2\theta)^{3/2}\sqrt{n^2-\sin^2\theta}}\right] \times (n^2-\sin^2\theta)^{1/2}$$

$$= \frac{\sin\theta}{n} \qquad \frac{1}{n} \times \sin\theta$$

$\sqrt{2}\times\sqrt{2} = 2$

$$= -\omega\sin\theta\left[\frac{\sqrt{(n^2-\sin^2\theta)}\times\sqrt{(n^2-\sin^2\theta)} - \cos^2\theta\times\cancel{\sqrt{n^2-\sin^2\theta}}}{(n^2-\sin^2\theta)^{3/2}\times\cancel{\sqrt{n^2-\sin^2\theta}}}\right]$$

$$= -\omega\sin\theta\left[\frac{(n^2-\sin^2\theta)-\cos^2\theta}{(n^2-\sin^2\theta)^{3/2}}\right]$$

$$= \frac{-\omega\sin\theta}{(n^2-\sin^2\theta)^{3/2}}\left[n^2-(\sin^2\theta+\cos^2\theta)\right]$$

$(\sin^2\theta+\cos^2\theta=1)$

$$\boxed{\frac{d(\omega_{pc})}{d\theta} = \frac{-\omega\sin\theta\,(n^2-1)}{(n^2-\sin^2\theta)^{3/2}}}$$

wkt $\alpha_{pc} = \dfrac{-\omega\sin\theta(n^2-1)}{(n^2-\sin^2\theta)^{3/2}}\times\omega$

$$\therefore\ \boxed{\alpha_{pc} = \frac{-\omega^2\sin\theta(n^2-1)}{(n^2-\sin^2\theta)^{3/2}}}$$

(*) $\boxed{\dfrac{-\omega^2\sin\theta}{n}}$

($\sin^2\theta$ is small as compare to n^2 $\therefore$ it may be neglected)

(Unity is small as comp to n^2 so will comp n)

(or) $= \dfrac{-\omega^2\sin\theta(n^2-1)}{n^3}$

$= -\omega^2\sin\theta\, C$

$(n^2)^{3/2}$ $\dfrac{n^4}{n^3} = n$

(*) — Sign indicates that the sense of angular acceleration of the rod is Reduce the angle ϕ.

Problem:

1. In a slider crank mechanism, the length of the crank and connecting rod are 100 mm and 400 mm respectively. The crank rotates uniformly @ 600 rpm clockwise when the crank has turned through 45° from the inner dead centre. Find, by analytical method.

1. velocity & acceleration of the slider, & 2. angular velocity & angular acceleration of the connecting Rod.

given data:

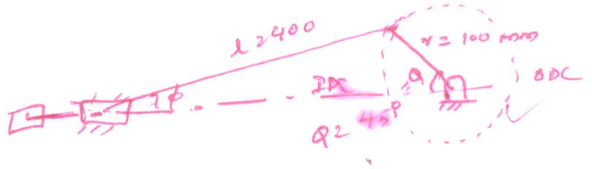
$\frac{100}{1000} = 0.1 m$

$r = 100 mm = 0.1 m$

$l = 400 mm = 0.4 m$

$N = 600 rpm$

$\theta = 45°$



piston = slider

To Find:

1. velocity & acceleration of the slider (V_p and a_p):

velocity of the slider is given by

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60}$$

Angular velocity $\omega = 62.83$ rad/sec

$$V_p = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

$$= 0.1 \times 62.83\left[\sin 45° + \frac{\sin(2 \times 45°)}{2 \times 4}\right]$$

$$n = \frac{l}{r} = \frac{0.4}{0.1} = 4$$

$$\boxed{V_p = 5.227 \text{ m/s}}$$ ANS

Acceleration of the slider is given by

$$a_p = \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= (62.83)^2(0.1)\left[\cos 45° + \frac{\cos(2 \times 45°)}{4}\right]$$

$$\boxed{a_p = 279.14 \text{ m/s}^2}$$ ans

2. Angular velocity & angular acceleration of the connecting rod (ω_{pc} and α_{pc}):

Angular velocity of the connecting rod is given by

$$\omega_{pc} = \frac{\omega \cos\theta}{n} = \frac{62.83 \times \cos 45^\circ}{4} = 11.1 \text{ rad/s}$$

$$\boxed{\omega_{pc} = 11.1 \text{ rad/s}}$$

(or) $\frac{\omega\cos\theta}{(n^2-\sin^2\theta)^{1/2}}$ $\sin^2\theta = (\sin\theta)^2$

Angular acceleration of the connecting rod is given by

$$\alpha_{pc} = \frac{\omega^2 \sin\theta}{n} = \frac{(62.83)^2 \sin 45^\circ}{4}$$

$$\boxed{\alpha_{pc} = 697.84 \text{ rad/s}^2}$$

(consider 2 stroke engine)

HW
2. A petrol engine has a stroke of 120 mm and connecting rod is 3 times the crank length. The crank rotates @ 1500 rpm clockwise direction. Determine: 1. velocity & acceleration of the piston & 2. angular velocity & angular acceleration of the connecting rod, when the piston has traveled one-fourth of its stroke from IDC.

(if dia not given means L consider like diameter)

given:

$L = 120 \text{ mm} = 0.12 \text{ m}$) radian/sec

$r = \frac{0.12}{2} = 0.06 \text{ m}$

~~$l = 3r$ (or) $n = \frac{l}{r} = 3$~~

1. $v_p = 8.235$ m/s
$a_p = 1046.83$ m/s²
$\omega_{pc} = 37.02$ r/s
$\alpha_{pc} = 5815.75$ r/s²

ans: $v_p = 8.235$ r/s
$a_p = 1046.83 \text{ m/s}^2$
$\omega_{pc} = 37.02$ r/sec
$\alpha_{pc} = 5815.75 \text{ rad/sec}^2$

Sol)

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(1500)}{60} = 157.08 \text{ rad/s}$$

It is given that the piston has traveled $1/4^{th}$ of its stroke from IDC

$$\theta = \frac{1}{4} \text{ of stroke from IDC} = \frac{1}{4} \times 180^\circ = 45^\circ$$

(2 stroke eng means 180°)

3. In a reciprocating engine mechanism, the crank and the connecting rod are 300 mm and 1 m long respectively and the crank rotates at a constant speed of 200 rpm. Determine analytically.

(i) the crank angle at which the maximum velocity occurs,
(ii) the maximum velocity of the piston.

given data : $\frac{300}{1000}$

$r = 300mm = 0.3 m$

$l = 1 m$

$N = 200$ rpm

To find :

Solution:

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94$ rad/s $\quad \boxed{\omega = 20.94 \text{ r/s}}$

obliquity ratio, $n = \frac{l}{r} = \frac{1}{0.3} = 3.33 \Rightarrow \boxed{n = 3.33}$

(i) crank angle at which the maxi velocity occurs:

θ = crank angle from IDC @ ~~with~~ which the maxi, velo occu

wkt: velocity of the piston:

$$v_p = \omega r \left[\sin\theta + \frac{\sin 2\theta}{2n} \right]$$

ω = velocity of

for maxi velocity of the piston,

$$\frac{dv_p}{d\theta} = 0 \text{ (or) } \frac{d}{d\theta}\left[\omega r \left(\sin\theta + \frac{\sin 2\theta}{2n}\right)\right] = 0 \quad \left(\frac{\sin 2\theta}{\cancel{2}} = \cos 2\theta \times 2\right)$$

$$\omega r \left[\cos\theta + \frac{\cos 2\theta \times \cancel{2}}{\cancel{n}\, 2n}\right] = 0 = \frac{0}{\omega r} = 0$$

LCM

$$\left(\because \cos 2\theta = 2\cos^2\theta - 1\right)$$

$$n\cos\theta + 2\cos^2\theta - 1 = 0$$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$2\cos^2\theta + n\cos\theta - 1 = 0$$

$2\cos^2\theta = 1 + \cos 2\theta$

$$\underset{a}{2\cos^2\theta} + \underset{b}{3.33\cos\theta} \underset{c}{- 1} = 0$$

$\boxed{\cos 2\theta = 2\cos^2\theta - 1}$

quadratic equ

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2(a)}$$

$\underset{a}{2}\cos^2\theta + \underset{b}{3.33}\cos\theta \underset{c}{-1} = 0$

$$\cos\theta = \frac{-3.33 \pm \sqrt{(3.33)^2 - 4(2\times -1)}}{2(2)}$$

$\frac{11.0889}{2.662}$

-8.88

3.6889

$\cos\theta = 0.259$

$1.75 = 0.38$

$\theta = \cos^{-1}(0.259)$

-1.5

$$\boxed{\theta = 74.9 \simeq 75^\circ}$$

(ii) Maximum velocity of the piston:

substituting the value $\theta = 75^\circ$ in the V_p eq, we get

$$V_{p(max)} = 20.94 \times 0.3\left[\sin 75^\circ + \frac{\sin(2\times 75)}{2\times 3.33}\right]$$

$V_p = \omega r\left[\sin\theta + \frac{\sin 2\theta}{n}\right]$

$$\boxed{V_{p(max)} = 6.54 \text{ m/s}}$$

Atew

4. the crank & connecting rod of a steam engine are 0.35 m and 1.55 m in length. The crank rotates @ 180 rpm. Clockwise. Determine the velocity and acceleration of the piston, when the crank is at 40° from the inner dead centre position. Also determine the position of the crank for zero acceleration of the piston. (θ=?) (θ)

give $r = 0.35$ m, $l = 1.55$ m, $N = 180$ rpm, $\theta = 40^\circ$

sol) $\omega = \frac{2\pi N}{60} = \frac{2\pi(180)}{60} = 18.85$ r/s, $n = \frac{l}{r} = \frac{1.55}{0.35} = 4.428$

(i) (V_p piston) $= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$

$a_p = \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$\cos 2\theta = \frac{1+\cos 2\theta}{2}$

(ii) $a_p = \omega^2 r\left[\cos\theta_1 + \frac{\cos 2\theta_1}{n}\right]$

$a_p = 0$ $\quad 0 = \omega^2 r\left[\cos\theta_1 + \frac{\cos 2\theta_1}{n}\right] \Rightarrow \frac{0}{\omega^2 r} = \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$ 2cm

$0 = n\cos\theta + \cos 2\theta = 0$ $\quad \cos 2\theta = 2\cos^2\theta - 1$

$[\cos 2\theta]$

$\underset{a}{2\cos^2\theta} + \underset{b}{\overset{(4.428)}{n}\cos\theta} \underset{c}{-1} = 0$

$\cos\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2(a)}$

KIOT - S. Rajeshkanna, AP/mech

Graphical method for velocity and acceleration of the Reciprocating Parts of Internal combustion Engines (IC)

Klien's construction:

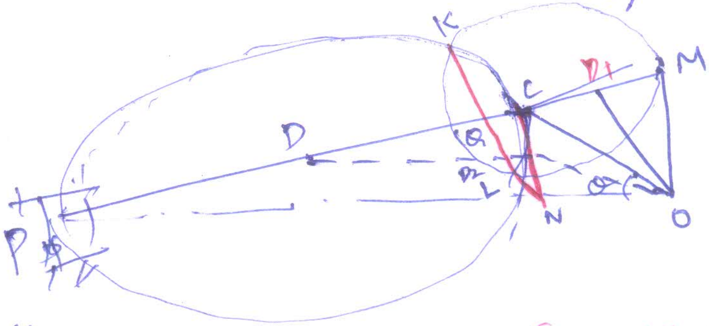
In klien's construction, the velocity and the acceleration diagrams are made on the configuration diagram itself.

Let us consider a slider crank mechanism OCP as shown in fig

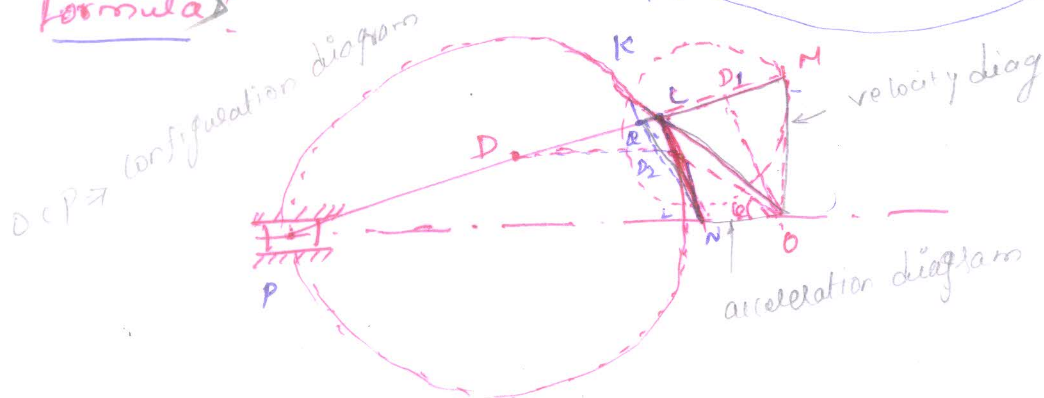
OC ⇒ is the crank,

CP ⇒ connecting rod

P ⇒ slider (or) piston



Formulas



(OCM - klien's velocity diag)

(~~OCNO~~ OCQN - klien's acceleration diagram)

Velocity of the piston P,

$$V_P = \omega \times OM \quad m/s$$

($\omega = 2\pi N/60$)

acceleration of piston P,

$$a_P = \omega^2 \times NO \quad m/s^2$$

Velocity of the mid-point of the connecting rod:

$$V_D = \omega \times OD_1 \quad m/s$$

acceleration of the mid-point of the connecting rod,

$$a_D = \omega^2 \times OD_2 \quad m/s^2$$

Angular velocity of the connecting rod

(velocity of connecting rod) $\rightarrow V_{PC} = \omega \times CM$

∴ Angular velocity of the connecting rod PC

$$\omega_{PC} = \frac{V_{PC}}{PC} \quad r/s$$

acceleration of connecting Rod

$$a'_{PC} = \omega^2 \times QN$$

∴ Angular acceleration of connecting rod is

$$\alpha_{PC} = \frac{a'_{PC}}{PC} \quad r/s^2$$

1. The crank and connecting rod of a reciprocating engine are 150 mm and 600 mm respectively. The crank makes an angle of 60° with the inner dead center and revolves ~~at~~ a uniform speed of 300 rpm. Find by klien's Construction. (or) Graphical method.

(i) velocity & acceleration of the piston

(ii) Velocity & acceleration of the mid-point of the connecting rod,

(iii) The angular velocity & angular acceleration of the connecting rod,

Given data:

$r = OC = 150\text{ mm}$, 1.5 cm

$l = PC = 600\text{ mm}$ 6 cm

$\theta = 60^\circ$ 100

$N = 300\text{ rpm}$

100 mm = 1 cm

150 mm = 1.5 cm 3 cm

600 mm = 6.0 cm ↑ 12 cm

2:1 scale

Sol)

$$\omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi \times 300}{60} = 31.42\text{ r/s}$$

see (or) draw diag from (13 Page)

By measurement to the chosen we get

$OM = 146\text{ mm} = 0.146\text{ m}$ $\qquad$ $CM = 75\text{ mm} = 0.075\text{ m}$

$QN = 130\text{ mm} = 0.130\text{ m}$ $\qquad$ $NO = 62\text{ mm} = 0.062\text{ m}$

(i) Velocity and acceleration of the piston:

wkt velocity of the piston P,

$$V_p = \omega \times OM \Rightarrow 31.416 \times 0.146$$

$$\boxed{V_p = 5.587\text{ m/s}}$$

acceleration of the piston P,

$$a_p = \omega^2 \times NO$$

$$= (31.416)^2 \times 0.062$$

$$\boxed{a_p = 61.19\text{ m/s}^2}$$

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(ii). velocity and accelelation of the mid-point of the connecting rod:

By measurement to the chosen scale, we get

$$OD_1 = 145\,mm = 0.145\,m$$

$\therefore$ velocity of D, $V_D = \omega \times OD_1$

$$= 31.416 \times 0.145$$

$$\boxed{V_D = 4.55\,m/s}$$

By measurement to the chosen scale, we get

$$OD_2 = 95\,mm = 0.095\,m$$

$\therefore$ accelelation of D, $a_D = \omega^2 \times OD_2$

$$a_D = (31.415)^2 \times 0.095$$

$$\boxed{a_D = 93.76\,m/s^2}$$

(iii) Angular velocity and angular accelelation of the connecting rod:

wkt velocity of connecting rod PC

$$V_{pc} = \omega \times CM = 31.416 \times 0.075$$

$$\boxed{V_{pc} = 2.3562\,m/s}$$

$\therefore$ Angular velocity of the connecting rod PC,

$$\omega_{pc} = \frac{V_{pc}}{pc} = \frac{2.3562}{0.6} = 3.927\,r/s$$

wkt accelelation of conn Rod:

$$a'_{pc} = \omega^2 \times QN = (31.416)^2 \times 0.130 = 128.305\,m/s^2$$

$\therefore$ angular accelelation of the connect Rod PC,

$$\alpha_{pc} = \frac{a'_{pc}}{pc} = \frac{128.305}{0.6} = 213.84\,rad/s^2$$

Force acting along the connecting Rod (F_Q) (or) Thrust in the connecting Rod

$$F_Q = \frac{F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}} \quad \text{(or)} \quad F_Q = \frac{F_P}{\cos\phi}$$

Thrust on the sides of cylinder walls: (or) pressure on slide bars!

$$\boxed{F_N = F_P \tan\phi} \text{ (or) } \quad \therefore F_Q = \frac{F_P}{\cos\phi} \quad \text{(or)}$$

$$F_Q = \frac{F_P}{\cos\phi}$$

Reaction b/w the piston & cylinder — normal reaction on the guide bars.

Crank-pin effort (F_T) $(\theta + \phi)$ (or) Tangential force on the crank pin

$$F_T = \frac{F_P}{\cos\phi} \times \sin(\theta + \phi) \quad \text{(or)} \quad F_T = F_Q \times \sin[\theta + \phi].$$

Thrust on crank shaft Bearing (F_B)

$$F_B = \frac{F_P}{\cos\phi} \times \cos(\theta + \phi)$$

Crank effort (or) Turning moment (or) Torque on the crankshaft (T):

enough

$$T = \frac{F_P \sin(\theta + \phi)}{\cos\phi} \times r \quad \text{(or)} \quad T = F_T \times r$$

$$T = F_P\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right] r$$

m_R = mass of the reciprocating parts (piston, gudgeon pin)

$W_R = m_R \cdot g$ = weight of the reciprocating parts

F_I = Inertial force (or) accelerating force of the reciprocating parts

F_L = Net load on the piston (or) force on the piston

F_P = piston effort

F_Q = Force acting along the connecting Rod

F_N = Thrust on the sides of the cylinder walls.

Formulas: forces on the reciprocating parts of an engine:

neglect the weight of the connecting Rod.

piston effort (F_P) (or) force on the piston

Net load on the gudgeon pin [F_P] (or) Rest pin

acceleration of the reciprocating parts (or) piston

$$a_R \text{ (or) } a_p = \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Inertia force of the reciprocating parts

$$F_I = m \cdot a$$

$$F_I = m_R \cdot a_R$$

$$= m_R \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

engines:

(a) For horizontal reciprocating engines

net thrust (or) piston effort (F_P) on the piston $= F_L \pm F_I$ (neglecting frictional resistance)

F_L → net load; F_I → Inertia force

$= F_L \pm F_I - R_F$ (considering frictional resis)

$R_F \Rightarrow$ ~~B~~ frictional resistance

(+) sign piston retarded (Back) ODC

(−) sign " accelerated [Front] IDC

For vertical reciprocating engines

piston effort $F_P = F_L - F_I + W_R$ (Neglecting frictional resistance)

$= F_L \pm F_I - R_F$ (considering fr " ")

To find net load on the piston (F_L): net Load on gudgeon pin

① for single cylinder single-acting engine:

$$F_L = \text{pressure} \times \text{Area}$$

$$= p \times \frac{\pi}{4} D^2$$

p in N/m²

$p = \frac{L}{A}$

D = diameter of the pistion in m

p = net pressure of steam (or) gas on the piston in N/m².

For single-cylinder double acting engine

$$F_L = p_1 A_1 - p_2 A_2$$

$$= p_1 A_1 - p_2 (A_1 - a)$$

cross sectional area of the piston $\frac{\pi}{4} d^2$

$(A_2 = A_1 - a)$

$a = \frac{\pi}{4} d^2$ cross sectional area of the piston

A_1 = cross-sectional area on the back side of the piston

A_2 = " " crank end side "

3. The lengths of crank and connecting rod of a horizontal engine are 200 mm & 1m respectively. The crank is rotating at 400 rpm. When the crank has turned through 30° from the inner dead centre, the difference of pressure b/w cover & piston rod is 0.4 N/mm². If the mass of the reciprocating parts is 100 kg & cylinder bore is 0.4 meters then calculate.

(i) inertial force (ii) force on piston
(iii) piston effort (4. thrust on the sides of the cylinder walls,
5. thrust in the connecting rod 6. crank effort (AU Nov/Dec 2005)

given data:

$r = 200\,mm = \frac{200}{1000} = 0.2\,m$

$l = 1\,m$

$N = 400\,rpm$

$\theta = 30^\circ$

$p_1 - p_2 = 0.4\,N/mm^2 = 0.4 \times 10^6\,N/m^2$

$m_R = 100\,kg$

Bore $(D) = 0.4\,m.$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 400}{60} = 41.89\,r/s$

$n = \frac{l}{r} = 1/_{0.2} = 5$

sol: (i) Inertia force (F_I):

$F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n} \right] \Rightarrow 100 \times (41.89)^2 \times 0.2 \left[\cos 30 + \frac{\cos 2(30)}{5} \right]$

formula

$F_I = 33.903\,KN$

2. net load on the piston (F_L): (force on the piston)

$A = \pi/_4 \times d^2 = \pi/_4 (0.4)^2$

$= \cancel{50.265\,K}$

$= 0.1256\,\cancel{mm}$

$A = 0.128\,m^2$

$\cancel{F_P = F_L - F_I}$

$F_L = (P_1 - P_2) A = (0.4 \times 10^6) \times 0.1256$

net load $\boxed{F_L = 50.265\,KN} = \boxed{50.265 \times 10^3\,N}$

$F_P = F_L - F_I$

$= 33.903 - 1776.42$

$\boxed{F_P = 26.498\,KN}$

3. Piston effort (F_P)

$$F_P = F_L - F_I = 50.265 \times 10^3 - 33.903 \times 10^3$$

$$\boxed{F_P = 16.36\ KN} \quad \boxed{F_P = 16.36 \times 10^3\ N}$$

4. Thrust on the side of the cylinder walls (F_N):

$$F_N = F_P \tan\phi \qquad \sin\phi = \frac{\sin\theta}{n} = \frac{\sin 30^\circ}{5}$$

$$= (16.36 \times 10^3)\tan(5.74^\circ) \qquad \sin\phi = 0.1$$

$$\boxed{F_N = 1.644\ KN} \quad \boxed{= 1.644 \times 10^3\ N} \qquad \phi = \sin^{-1}(0.1)$$

$$\boxed{\phi = 5.74^\circ}$$

5. Thrust in the connection rod (F_Q)

$$F_Q = \frac{F_P}{\cos\phi} = \frac{16.36 \times 10^3}{\cos(5.74)} = 16.444\ KN = 16.444 \times 10^3\ N$$

6. crank effort (F_T)

$$F_T = \frac{F_P \sin(\theta + \phi)}{\cos\phi} \times r$$

$$= \frac{16.36 \times 10^3 \times \sin(30 + 5.74)}{\cos(5.74)} \times 0.2$$

$$\boxed{F_T = 1920.83\ N\text{-}m}$$

2-20 H.W A horizontal steam engine running @ 210 rpm has a bore of 190 mm & stroke of 350 mm. the piston rod is 20 mm in diameter & connecting rod length is 950 mm. The mass of the reciprocating parts is 8 kg & the frictional resistance is equivalent to a force of 350 N.

Determine the following when the crank is @ 115° from IDC. the mean pressure being 14500 N/m² on the cover side & 100 N/m² on the crank side

1. thrust on the connecting rod (~~force~~) 2. thrust on the cylinder walls, 3. load on the bearings 4. Turning moment on the crankshaft.

1. $F_Q = \frac{F_P}{\cos\phi}$ $= F_P = F_L - F_I - R_F$ | 2. $F_N = F_P \tan\phi$ | 3. $F_B = F_Q \cos(\theta + \phi)$

$\sin\phi = \frac{\sin\theta}{n}$

4. $F_T \times r$

(17) A vertical petrol engine 150 mm dia & 200 mm stroke has a connecting rod 350 mm long. The mass of the piston is 1.6 kg & the engine speed is 1800 rpm. On the expansion stroke with crank angle 30° from top dead centre, the gas pressure 750 kN/m². Determine the net thrust on the piston, & also turning moment on crankshaft.

given data:

$D = 150\,mm = 0.15\,m$

$L = 200\,mm = 0.2\,m \Rightarrow r = 0.2/2 = 0.1\,m$

$l = 350\,mm = 0.35\,m$

$m = 1.6\,kg$

$N = 1800\,rpm$

$\theta = 30^\circ$

$p = 750\,kN/m^2 = 750 \times 10^3\,N/m^2$

$n = \frac{l}{r} = \frac{0.35}{0.1} = 3.5$

$\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 30^\circ}{3.5}$

$\boxed{\phi = 8.21^\circ}$

sol

1. net thrust on the piston is (piston effort)

F_P (or) $F_P = F_L - F_I + W_R$

load on the piston $F_L = p \times A = p \times \frac{\pi}{4} D^2 = 750 \times 10^3 \times \left(\frac{\pi}{4} \times (0.15)^2\right)$

$\boxed{F_L = 13523.6\,N}$

(Inertia force on the piston) $F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$ $\boxed{F_I = ma}$

$= 1.6\,(188.49)^2 \times 0.1 \left[\cos 30^\circ + \frac{\cos 2(30^\circ)}{3.5}\right]$

$\boxed{F_I = 5735.05\,N}$

Turning moment

$T = \left[\frac{F_P}{\cos\phi} \sin(\theta + \phi)\right] r$

$= 0.35 \times \frac{7534.23}{\cos(8.21)} \sin(30 + 8.2\ldots)$

$= \_\_\_\_$ ANS

for vertical engine force acts becoze of mass piston

$W_R = m_R \cdot g = 1.6 \times 9.81 = 15.696\,N$

$\boxed{W_R = 15.696\,N}$

net thrust on the piston (piston effort)

F_P (or) $F_P = F_L - F_I + W_R = 13253.6 - 5735.05 + 15.696$

$\boxed{F_P = 7534.23\,N}$

(H.W) The ratio of the connecting rod length to crank length for a vertical petrol engine is 4:1. The bore/stroke is 80/100 mm & mass of the reciprocating part is 1 kg. the gas pressure on the piston is 0.5 N/mm2 when it has moved 10mm from TDC on its power stroke. Determine the net load on the gudgeon pin. the engine runs @ 1800 rpm. At what engine speed will the (pisto) load be zero?

given

$n = \frac{l}{r} = 4$; $\frac{D}{L} = 80/100$ mm $D = 80$ mm $= 0.08$ m

$L = 100$ mm $= 0.1$ m (or) $r = \frac{0.1}{2} = 0.05$ m

$m_R = 1$ kg , $P = 0.7$ N/mm² $= 0.7 \times 10^6$ N/m²

$x = 10$ mm $= 10 \times 10^{-3}$ m (or) $\frac{10}{1000} = 0.01$ m

$N = 1800$ rpm

$N = ?$ (speed)

$F_P = ?$ (piston effort)

sol

$$x = r\left[(1 - \cos\theta) + \frac{\sin^2\theta}{2n}\right]$$ using this formula will find $\theta = ?$

1. net load on the gudgeon pin (piston effort) F_P

$$F_P = \underbrace{F_L}_{\text{load on piston}} - \overbrace{F_I}^{\text{Inertia force}} + W \qquad W = mg$$

2. engine speed @ which F_P is zero:

$$F_P = F_L - F_I + W$$

$$F_L - F_I + W = 0$$

$$F_L + W = F_I$$

$$F_L + W = m\omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$\omega = $ —? Ans

$\omega = \frac{2\pi N}{60} = N = $ —? Ans

1. A three cylinder single acting engine has its cranks set equally at 120° & it runs at 600 rpm. The torque-crank angle diagram for each cycle is a triangle for the power stroke with a maxi torque of 90 N-m at 60° from dead centre of corresponding crank. The torque on the return stroke is sensibly zero. Determine

1. power developed (P) 2. Coefficient of fluctuation of speed (C_s) if the mass of the flywheel is 12 kg has a radius of gyration of 80 mm. 3. Coefficient of fluctuation of Energy. 4. maxi angular acceleration of the flywheel.

(580) KU

Given:

$N = 600$ rpm

$T_{max} = 90$ N-m

$m = 12$ kg

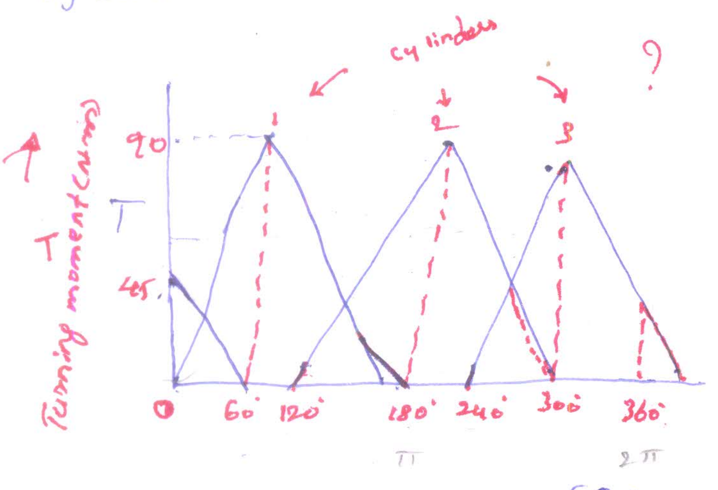
$k = 80$ mm $= 0.08$ m

To Find:

1. $P = \quad P = \frac{2\pi NT}{60}$
2. $C_s =$
3. $C_E = \frac{\text{maxi flu of Energy}}{\text{workdone/cycle}}$
4. $\alpha_\theta =$

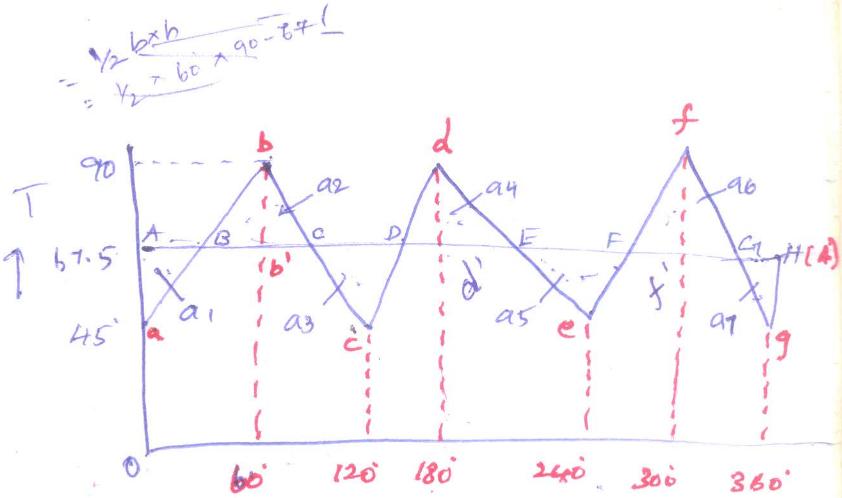
Sol

The torque-crank angle diagram for the individual cylinders is Fig (1) & the Resultant torque-crank angle diagram for the three cylinders Fig (2).



crank angle (θ)

(1)



crank angle → θ

(2)

1. Power developed: $P = \frac{2\pi N T_{mean}}{60}$

WKT workdone / cycle, Area of one triangle $= \frac{1}{2} \times b \times h \Rightarrow \frac{1}{2} \times 180 \times 90$ (or) π

$w/cy = T_{mean} \times \theta$

w/cy = Area of three triangles $= 3 \times \frac{1}{2} \times \pi \times 90 = 424.115$ N-m

($\pi = 180°$)

work done / cycle $= T_{mean} \times \theta$ [crank angle / cycle]

(or) Workdo/cy $= P \times 60$

$T_{mean} = \frac{\text{work done / cycle}}{\text{Crank angle / cycle}} = \frac{424}{2\pi} = 67.5 \text{ N-m}$

360 ? $(2\pi = 360°)$ ⊗

$\omega = \frac{2\pi N}{60}$ rad/s

∴ Power developed = $T_{mean} \times \omega$

(or)

$P = \frac{2\pi N T_{mean}}{60} = \frac{2\pi \times 600 \times 67.5}{60}$

$\boxed{P = 4239 \text{ W} \;(\text{or})\; 4.239 \text{ KW}}$

10^3

2. **Coefficient of fluctuation of speed : (C_s) :**

First of all, let us find the max fluctuation of energy (ΔE)

From fig (2)

a_1 = Area of triangle AaB

$= \frac{1}{2} \times AB \times Aa$

$= \frac{1}{2} \times \frac{\pi}{6} \times (67.5 - 45)$ (or) 30° (∴ $AB = 30° = \frac{\pi}{6}$ rad) ← 0.5

$\boxed{a_1 = 5.89 \text{ N-m} = a_7}$

(or) $30° \times \frac{\pi}{180} = \frac{\pi}{6}$ rad) ← 0.5

$\boxed{a_1 = a_7}$ ⊗

a_2 = Area of triangle Bbc

$= \frac{1}{2} \times BC \times bb'$

$= \frac{1}{2} \times (60° \times \frac{\pi}{180}) \times (90 - 67.5)$ (∴ $BC = 60° = (60 \times \frac{\pi}{180}$ = rad))

$= 11.78$ N-m

−11.78 + 47.12

$\boxed{a_2 = a_3 = a_4 = a_5 = a_6}$ ⊗

Now, let the total energy @ A = E,

Crank ~~angle~~ position	Flywheel Energy	
A	E	$= E$
B	$E - 5.89$	(min) $= E - 5.89$
C	$E - 5.89 + 11.78$	(max) $= E + 5.89$
D	$E - 5.89 + 11.78 - 11.78$	$= E - 5.89$
E	$E - 5.89 + 11.78 - 11.78 + 11.78$	(max) $= E + 5.89$
F	$E - 5.89 + 11.78 - 11.78 + 11.78 - 11.78$	(min) $= E - 5.89$
G	$E - 5.89 + 11.78 - 11.78 + 11.78 - 11.78 + 5.89$	$= E + 0$

maximum fluctuation of Energy:

$$\Delta E = \text{max Energy} - \text{min Energy}$$

$$= (E + 5.89) - (E - 5.89)$$

$$\boxed{\Delta E = 11.78 \text{ N-m}}$$

$$\Delta E = I\omega^2 C_s$$

$$11.78 = \overset{(12)\times(0.08)^2}{mk^2} \times (62.8)^2 \times C_s$$

$$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 600}{60}$$

$$\omega = 62.8 \text{ rad/s}$$

$$C_s = \frac{11.78}{302.88}$$

$$C_s = 0.038 \times 100$$

$$\boxed{C_s = 3.889\ \%}$$

3. Coefficient of fluctuation of Energy:

$$C_E = \frac{\text{max. fluctuation of energy}}{\text{workdone/cycle}} = \frac{11.78}{424} = 0.0277$$

$$C_E = 0.02778 \times 100$$

$$\boxed{C_E = 2.778\ \%}$$

4. maximum angular acceleration of the flywheel :

α = maxi angular acceleration of the fly wheel

$T = I\alpha$

T = inertia torque $= I\alpha$

$T = T_{max} - T_{mean}$

$T_{max} - T_{mean} = I\alpha = mk^2\alpha$

Inertia of the Body opposes this externally applied torque.

$90 - 67.5 = 12 \times (0.08)^2 \times \alpha$

$\alpha = \dfrac{90 - 67.5}{0.077}$

$\dfrac{kg\text{-}m}{kg\text{-}m}$

$$\boxed{\alpha = 292 \text{ rad/s}^2}$$

(2) The turning moment diagram for a four stroke gas engine may be assumed for simplicity to be represented by four triangles, the areas of which from the line of zero pressure are as follows:

Suction stroke $= 0.45 \times 10^{-3}\, m^2$; compression stroke $= 1.7 \times 10^{-3}\, m^2$; Expansion stroke $= 6.8 \times 10^{-3}\, m^2$, Exhaust stroke $= 0.65 \times 10^{-3}\, m^2$. each m^2 of area represents 3 MN-m of energy.

Assuming the resisting torque to be uniform, find the mass of the rim of a flywheel required to keep the speed b/w 202 & 198 rpm. The mean radius of the rim is 1.2 m. also find the crank positions for the max & min speeds.

Given data :

$a_1 = 0.45 \times 10^{-3}\, m^2$

$a_2 = 1.7 \times 10^{-3}\, m^2$

$a_3 = 6.8 \times 10^{-3}\, m^2$

$a_4 = 0.65 \times 10^{-3}\, m^2$

$N_1 = 202$ rpm

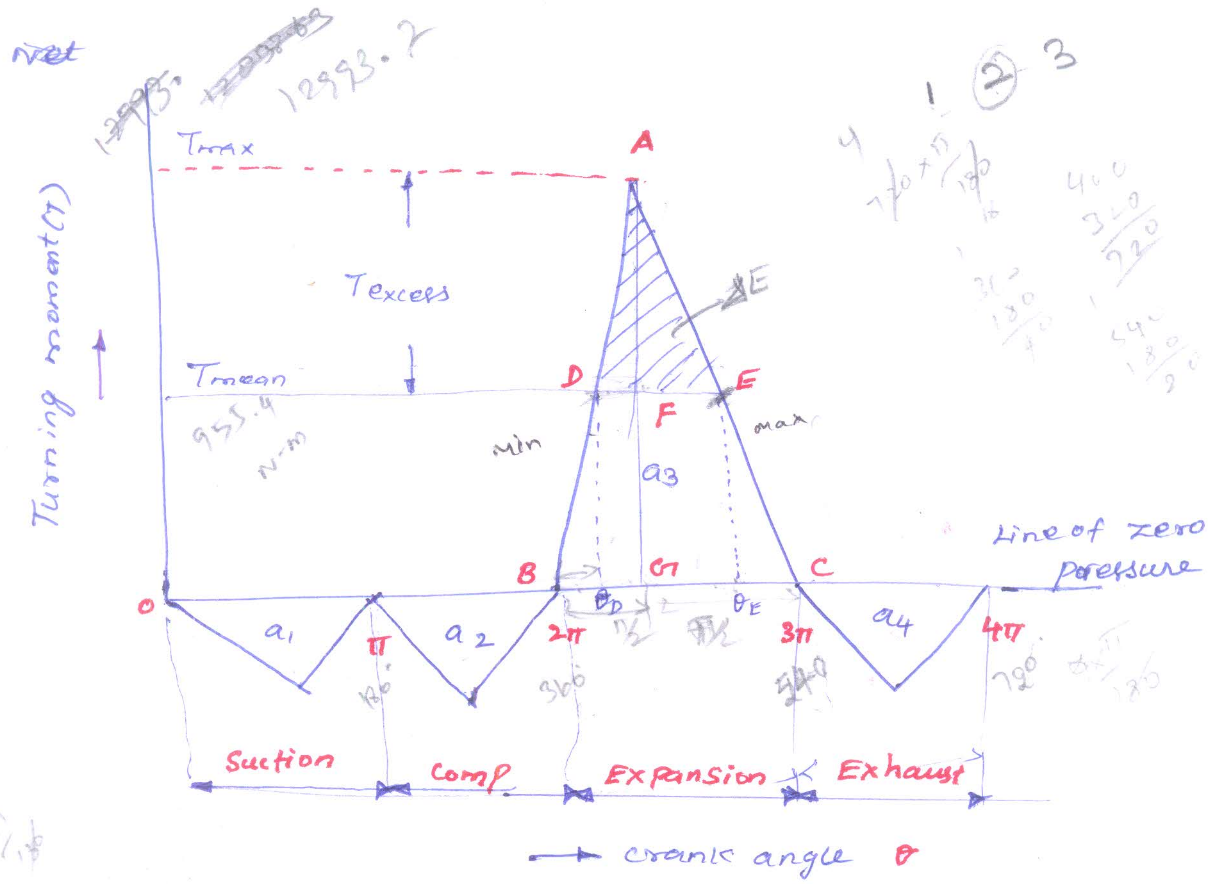
$N_2 = 198$ rpm

$R = 1.2$ m

To Find /

1. mass of the rim (m)
2. Crank position (θ_D, θ_E)

2. The areas below the zero line of pressure are taken as negative while the areas above the zero line ~~of~~ pressure are taken as positive.



$$\text{net area} = a_3 - (a_1 + a_2 + a_4)$$

$$= 6.8\times10^{-3} - (0.45\times10^{-3} + 1.7\times10^{-3} + 0.65\times10^{-3})$$

$$= 4\times10^{-3}\ m^2$$

since the energy scale is $1\ m^2 = 3\ MN\text{-}m = 3\times10^6\ N\text{-}m$

$$\therefore \text{net work done/cycle} = 4\times10^{-3} \times 3\times10^6 = \underline{\underline{12\times10^3}}\ N\text{-}m$$

$$\text{work done/cycle} = T_{mean} \times \theta$$

($\because$ from fig $\theta = 4\pi$)

$$12\times10^3 = T_{mean} \times 4\pi$$

$$FG = T_{mean} = \frac{12\times10^3}{4\pi} = 955.41\ Nm$$

FG (or) $\boxed{T_{mean} = 955.41\ N\text{-}m}$

work done during expansion stroke

$= a_3 \times$ Energy Scale

$= 6.8 \times 10^{-3} \times 3 \times 10^{6}$

$= 20.4 \times 10^{3}$ N-m ——(1)

Also, work done during expansion stroke

= Area of triangle ABC

$= \frac{1}{2} \times BC \times AG$

$= \frac{1}{2} \times \pi \times AG = 1.57 \times AG$ ——(2)

$20.4 \times 10^{3} = 1.57 \times AG$

(BG) $AG = \frac{20.4 \times 10^{3}}{1.57}$, $\boxed{AG = 12993.63 \text{ Nm}}$

~~now from similar triangles~~ ADE & ABC

∴ excess torque

$T_{excess} = AF = AG - FG$ BG - ?

$= 12993.63 - 955$

AF (or) $\boxed{T_{excess} = 12038.63 \text{ N-m}}$

now from similar triangles ADE & ABC

$\frac{min}{max} = \frac{min}{max} \Rightarrow \frac{DE}{BC} = \frac{AF}{AG}$

$\frac{DE}{\pi} = \frac{12038.63}{12993.63}$

$\boxed{DE = 2.909 \text{ rad}}$

we know that the maximum fluctuation of Energy

$$\Delta E = \text{Area of } \Delta ADE$$

$$= \frac{1}{2} \times DE \times AF$$

$$= \frac{1}{2} \times 2.909 \times 12038.63$$

$$\boxed{\Delta E = 17511.498 \text{ N-m.}}$$

mass of the rim of a fly wheel :

K = R

$$\Delta E = I \omega^2 C_s$$

$$I = mk^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.933$$

$$C_s = \frac{N_1 - N_2}{N} = \frac{202 - 198}{200} = 0.02$$

$$N = \frac{N_1 + N_2}{2} = \frac{202 + 198}{2}$$

$$\boxed{N = 200}$$

$$17511.498 = m \times (1.2)^2 \times (20.933)^2 \times 0.02$$

$$\boxed{m = 1388.21 \text{ kg}}$$

crank position for the max & min speeds :

wkt the speed of the flywheel is min @ point D & max at point E. Let θ_D and θ_E = crank angles from IDC for the min & max speed.

From similar triangles DAF & ~~ABG~~ BAG

$$\frac{min}{max} = \frac{min}{max}$$

$$\frac{DF}{BG} = \frac{AF}{AG} \Rightarrow \frac{DF}{\frac{\pi}{2}} = \frac{12038.63}{12993.63}$$

$$\boxed{DF = 1.454 \text{ rad}}$$

$$\theta_D = BG - DF$$

$$= \frac{\pi}{2} - 1.454$$

$$\boxed{\theta_D = 0.1161 \text{ rad}}$$

$$\theta_D = \frac{0.1161}{\frac{\pi}{180}} = 6.65$$

$$\boxed{\theta_D = 6.65^\circ}$$

From similar triangles AFE & AGC

$$\frac{min}{max} = \frac{min}{max}$$

$$\frac{FE}{GC} = \frac{AF}{AG} \Rightarrow \frac{FE}{\pi/2} = \frac{12038.63}{12993.63}$$

$$FE = 1.4553 \text{ rad}$$

$$\theta_E = BG + FE$$

$$= \pi/2 + 1.4553$$

$$\boxed{\theta_E = 3.026 \text{ rad}}$$

$$\frac{3.026 \times}{\frac{\pi}{180}} \Rightarrow \theta_E$$

$$\boxed{\theta_E = 173.38^\circ}$$

Dimensions of the flywheel rim:

1. hoop stress $\sigma = \rho v^2$

2. Peripheral velocity $(v) = \sqrt{\frac{\sigma}{\rho}}$

3. mass of the rim (m) = volume x density

$$m = v \times \rho$$

$$m = \pi D A \times \rho$$

4. Area of the rim (A):

$$A = \frac{m}{\pi D \rho}$$

density = $\frac{mass}{volume}$

linear velocity of flywheel

$$v = R\omega = \frac{\pi D N}{60}$$

$$\omega = \frac{2\pi N}{60} \times R$$

$\omega \times R$

R = mean radius of rim in meters

D = " dia " "

A = cross-sectional area of rim in m^2

ρ = density of rim mtl in kg/m^3

N = Speed of the flywheel in rpm

ω = angular velocity of flywheel $/s^2$

$v = R\omega = \frac{\pi D N}{60}$ linear velocity of the

σ = Tensile stress (or) hoop stress N/m^2.

1. A Steam engine runs at 150 rpm. Its turning moment diagram gave the following area measurements in mm^2 taken in order above & below the mean torque line 500, −250, 270, −390, 190, −340, 270, −250. The scale for the turning moment is 1mm = 500 N-m & for crank angle is 1mm = 5°

If the fluctuation of speed is not to exceed ±1.5% of the mean, determine a suitable diameter & cross-section of the rim of the flywheel assumed with axial dimension (width of the rim) equal to 1.5 times the radial dimension. The hoop stress is limited to 3 Mpa & the density of the material of the flywheel is 7500 kg/m^3.

Given

$N = 150$ rpm

$C_s = \pm 1.5\% = 3\% = 0.03$

$b = 1.5 \times$ radial dimension $= 1.5 \times t$

$\sigma = 3$ Mpa $= 3 \times 10^6$ N/m^2

$\rho = 7500$ kg/m^3

To Find

1. D (diameter)
2. cross-section of the flywheel (b×t)

Sol

1. Diameter of the rim (D)

wkt hoop stress $\sigma = \rho v^2$

$$3 \times 10^6 = 7500 \times v^2$$

$$\boxed{v = 20 \text{ m/s}}$$

wkt $v = \frac{\pi D N}{60}$

$$20 = \frac{\pi \times D \times 150}{60}$$

Dia of the flywheel rim } $D = 2.546$ m

2. Cross-section of the flywheel rim (b & t)

First at all, let us find the max fluctuation of energy

The turning moment scale:

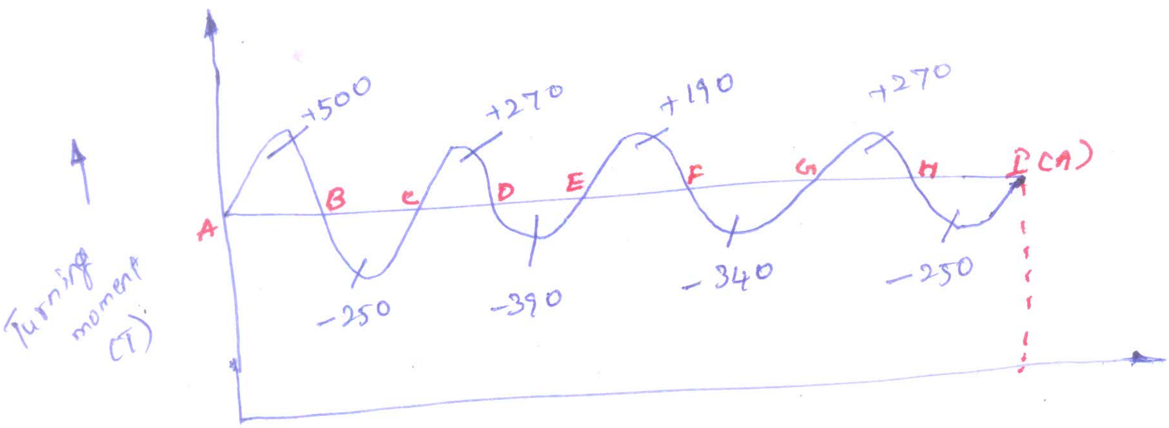
Turning moment 1 mm = 500 N-m &

crank angle 1 mm = 5°

$1\,mm^2$ on the turning moment diagram $= 500 \times \left(5^\circ \times \frac{\pi}{180}\right)^2$

$= 43.63$ N-m

turning moment diagram



(all areas are in mm^2)

crank position	flywheel energy (mm^2)	
A	E	= E
B	E +500	= E +500
C	E +500 -250	= E +250
D	E +500 -250 +270	(max) = E +520
E	E +500 -250 +270 -390	= E +130
F	E +500 -250 +270 -390 +190	= E +320
G	E +500 -250 +270 -390 +190 -340	(min) = E -20
H	E +500 -250 +270 -390 +190 -340 +270	= E +250
I	E +500 -250 +270 -390 +190 -340 +270 -250	= E +0

ΔE = max Energy − min Energy

$= (E+520) - (E-20)$

$= 540\ mm^2$

$\boxed{\Delta E = 540 \times 43.63\ N\text{-}m}$ $\quad (\because 1\ mm^2 = 43.63\ N\text{-}m)$

$\Delta E = 23560.2\ N\text{-}m$

$\Delta E = I\omega^2 C_s$

$\because I = mk^2$

~~23560~~ 2 = ~~m × (20~~

$\Delta E = mR^2\omega^2 C_s = mv^2 C_s$ $\quad \boxed{K = R}$ $\quad v = R\omega$

$v = \frac{\pi D N}{60}$

$\omega^2 = \left(\frac{2\pi N R}{60}\right)^2$

$23560.2 = m \times (20)^2 \times 0.03$

$\boxed{m = 1963.35\ kg}$ ✓

wkt . mass of the flywheel rim (m)

density = $\frac{mass}{volume}$ $\quad$ m = volume × density

$m = \pi D A \times \rho$

$1963.35 = \pi \times 2.546 \times A \times 2700$

$\boxed{A = 0.0341\ m^2}$

wkt $\quad A = b \times t$ (width × thickness)

$0.0341 = (1.5 \times t) \times t$

$0.0341 = 1.5\, t^2$

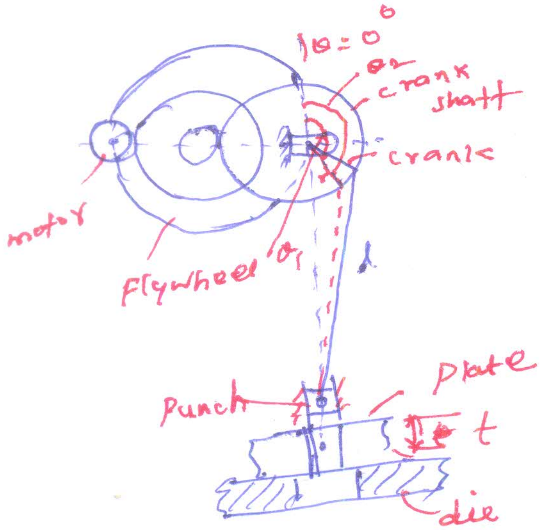
0.1507×1000

thickness of the rim $\boxed{t = 0.1507\ m \quad (or)\quad 150.7\ mm}$

width of the rim (b) = $1.5 \times t \Rightarrow 1.5 \times (0.1507)$

$\boxed{b = 0.226\ m \quad (or) \quad 226\ mm}$

Flywheel in Punching Press:



663 KU (HW)

1. A punching press is driven by a constant torque electric motor. The press is provided with a flywheel that rotates at max speed of 225 rpm. The radius of gyration of the flywheel is 0.5 m. The press punches 720 holes per hour. each punching operation takes 2 second & requires 15 KN-m of energy. Find the power of the motor & min mass of the flywheel if speed of the same is not to fall below 200 rpm.

given:

$N_1 = 225$ rpm

$K = 0.5$ m

hole punched = 720 per hr

$E_1 = 15$ KN-m $\Rightarrow 15\times10^3$ N-m

$N_2 = 200$ rpm

To find

1. P = ? (Power)

2. minimum mass of the fly wheel.

sol

Power of the motor:

wkt Power the total energy required per second

= Energy required / hole × No. of holes / sec

$= 15\times10^3 \times 720/3600$

$= 3000$ N-m/s $\qquad (\because 1 \text{ N-m/s} = 1\text{W})$

$\therefore$ Power of the motor $= 3000$ W $= 3$ kW

ΔE = max Energy − min Energy

$= (E+520) - (E-20)$

$= 540\ mm^2$

$\boxed{\Delta E = 540 \times 43.63\ N\text{-}m}$ $(\because 1\ mm^2 = 43.63\ N\text{-}m)$

$\Delta E = 23560.2\ N\text{-}m$

$\Delta E = I\omega^2 C_s$

$\because I = mk^2$

~~23560~~.2 = ~~$m \times (20$~~

$\Delta E = mk^2\omega^2 C_s = mv^2 C_s$

$\boxed{K = R}$ $v = R\omega$

$v = \frac{\pi D N}{60}$

$\omega^2 = \left(\frac{2\pi N}{60}\right)^2$

$23560.2 = m \times (20)^2 \times 0.03$

$\boxed{m = 1963.35\ kg}$ ✓

wkt mass of the flywheel rim (m)

density $= \frac{mass}{volume}$ m = volume × density

$m = \pi D A \times \rho$

$1963.35 = \pi \times 2.546 \times A \times 2700$

$\boxed{A = 0.0341\ m^2}$

wkt $A = b \times t$ (width × thickness)

$0.0341 = (1.5 \times t) \times t$

$0.0341 = 1.5 t^2$

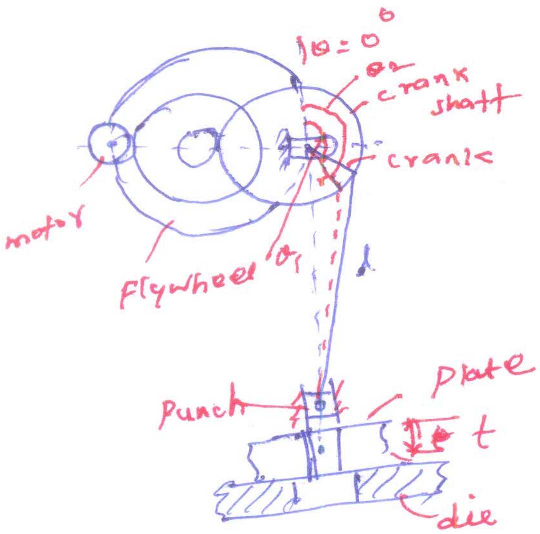
0.1507×1000

thickness of the rim $\boxed{t = 0.1507\ m \text{ (or) } 150.7\ mm}$

width of the rim (b) $= 1.5 \times t \Rightarrow 1.5 \times (0.1507)$

$\boxed{b = 0.226\ m \text{ (or) } 226\ mm}$

Flywheel in Punching Press:



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given:

$N_1 = 225$ rpm

$K = 0.5$ m

hole punched = 720 per hr

$E_1 = 15$ kN-m $\Rightarrow 15 \times 10^3$ N-m

$N_2 = 200$ rpm

To find

1. P = ? (Power)

2. minimum mass of the fly wheel.

sol

Power of the motor:

wkt ~~Power~~ the total energy required per second

$= $ Energy required / hole $\times$ No. of holes / sec

$= 15 \times 10^3 \times 720 / 3600$

$= 3000$ N-m/s $\quad (\because 1 \text{ N-m/s} = 1\text{W})$

$\therefore$ Power of the motor $= 3000$ W $= 3$ kW minimum mass of the flywheel:

m = min mass of the flywheel

since each punching operation takes 2 seconds, therefore energy supplied by the motor in 2 seconds.

$$E_2 = 3000 \times 2 = 6000 \text{ N-m}$$

∴ Energy to be supplied by the flywheel during punching (or) max fluctuation of energy

$$\Delta E = E_1 - E_2$$

$$= 15 \times 10^3 - 6000$$

$$\boxed{\Delta E = 9000 \text{ N-m}}$$

~~mean speed~~:

$$N = \frac{N_1 + N_2}{2} = \frac{225 + 200}{2}$$

$$= 212.5 \text{ rpm}$$

$$\Delta E = I\omega^2 C_s$$

$$\Delta E = mk^2 \omega^2 C_s$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 212.5}{60}$$

$$\omega = 22.24 \text{ rad/s}$$

$$9000 = m \times (0.5)^2 \times (22.24)^2 \times C_s$$

$$m = \frac{9000}{(0.5)^2 \times 494.69 \times 0.1176}$$

~~494~~ 69

$$C_s = \frac{N_1 - N_2}{N} = \frac{225 - 200}{212.5}$$

$$C_s = 0.1176$$

~~m = 61.856~~

$$\boxed{m = 618.8167 \text{ kg}}$$

dimention of the fly wheel :

1. hoop stress $\sigma = \rho v^2$

2. peripheral velocity $v = \sqrt{\frac{\sigma}{\rho}}$

3. mass of the rim m : volume x density $\rho = \frac{m}{v}$

$$m = \pi D A \times \rho$$

$$A = \frac{m}{\pi D \rho}$$

59

1. The turning moment diagram of a four stroke engine may be assumed for the sake of simplicity to be represented by four triangles in each stroke. The areas of these triangles

suction stroke = $5 \times 10^{-5}\ m^2$, compression stroke = $21 \times 10^{-5}\ m^2$;
Expansion stroke = $85 \times 10^{-5}\ m^2$, exhaust stroke = $8 \times 10^{-5}\ m^2$.

All the areas excepting expansion stroke are negative. each m^2 of area represents 14 MN-m of work.

Assuming the resisting torque to be constant. determine the moment Inertia of the flywheel to keep the speed b/w 98 rpm to 102 rpm. also find the size of a rim - type - flywheel based on the min mtl criterion. given that density of flywheel mtl is 8150 kg/m³. the allowable tensile stress of the flywheel mtl is 7.5 mpa the rim cross-section is rectangle, on side being four times the length of the other.

given:

$a_1 = 5 \times 10^{-5}\ m^2$

$a_2 = 21 \times 10^{-5}\ m^2$

$a_3 = 85 \times 10^{-5}\ m^2$

$a_4 = 8 \times 10^{-5}\ m^2$

$N_1 = 102\ rpm$

$N_2 = 98\ rpm$

$\rho = 8150\ kg/m^3$

$\sigma = 7.5\ Mpa = 7.5 \times 10^6\ N/m^2$

To Find

moment of Inertia (I)

Size of the flywheel ($b \times t$)

$\boxed{\Delta E = 10160\ N\text{-}m}$

$N = \frac{N_1 + N_2}{2} = \frac{102 + 98}{2} = 100\ rpm$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60}$

$\boxed{\omega = 10.47\ r/s}$

$C_s = \frac{N_1 - N_2}{N} = \frac{102 - 98}{100}$

$\boxed{C_s = 0.04}$

I

$\Delta E = I \omega^2 C_s$

$10160 = I \times (10.47)^2 \times 0.04$

$\boxed{I = 2317\ kg\text{-}m^2}$

$\boxed{I = \frac{\Delta E}{\omega^2 C_s}}$

Size of flywheel:

$b = 4t$

wkt

hoop stress (σ)

$\sigma = \rho v^2$

$7.5 \times 10^6 = 8150\ v^2$

$v^2 = \frac{7.5 \times 10^6}{8150}$

$v^2 = 920$

$\boxed{v = 30.3\ m/s}$

$\frac{N/m^2}{kg/m^3}$ $\frac{N}{m^2} \frac{m^3}{kg}$

$\frac{N\,m}{kg}$

$\frac{kg}{N} = 9N\,m/kg$

$\frac{N}{kg}\,m$

$V = \frac{\pi D N}{60}$ $\qquad V = \frac{m}{T}$ $\qquad V = \frac{\pi D N}{60}$ $\quad \omega = \frac{2\pi N}{60}$

5m.

$\cancel{D} = \frac{V \times 60}{\pi N} = \frac{30.3 \times 60}{\pi \times 100}$

$\boxed{D = 5.786 \text{ m}}$

m/s

rev/min / sec.

m/rev.

$\Delta E = I\omega^2 C_s$

$= m k^2 \omega^2 C_s$

$10160 = m \times (2.893)^2 \times 0.04 \times (10.47)^2$

$10160 = m \times 8369 \times 0.04$

$\Delta E = I\omega^2 C_s$

$10160 = m k^2 \omega^2 C_s$ $\qquad k = R \qquad \frac{D}{2} = R$

$I = m k^2$

$k = R$

$10160 = m \times (2.893)^2 \times (10.47)^2 \times 0.04$

N_1, N_2

$C_s = \frac{N_1 - N_2}{N}$

$N = \frac{N_1 + N_2}{2}$

$0.02 \times$

$\boxed{m = 276.86 \text{ kg}}$

mass = volume x density

$\rho = \frac{m}{V} \qquad \therefore V = \pi D A$

$m = \pi D A \times \rho$

$\rho = \frac{m}{\pi D A}$

$A = \frac{m}{\pi D \rho} = \frac{276.86}{\pi \times 5.786 \times 8150}$

$A = \frac{m}{\pi D \rho}$ $\quad$ 12.8

$\frac{kg \times N}{m \times kg/m^2}$

$= \frac{kg \times \cancel{N}}{m \times N/m^2}$

$A = 1.8697 \times 10^{-3}$ m²

$= \frac{kg \times N}{\cancel{m} \times kg / m^2}$

$= m^2$

$A = b \times t$

5t

$1.8697 \times 10^{-3} = 4t \times t$

$b = 4t$

$1.869 \times 10^{-3} = 4t^2$

$\frac{1.869 \times 10^{-3}}{4} = t^2$

$4.6725 \times 10^{-4} = t^2$

$\boxed{t = 0.0216 \text{ m}}$

$\boxed{t = 21.6 \text{ mm}}$

$b = 4t \Rightarrow 4 \times 0.0216$

$\boxed{b = 0.086 \text{ m}}$

$\boxed{b = 86.46 \text{ mm}}$

Turning Moment diagrams:

4. design of ~~the~~ flywheel.

The turning moment diagram is the graphical representation of the turning moment (T) for various positions of the crank (θ).

~~us~~ (write) T - formula
1. To determine work done per cycle & power developed
2. mean Torque & fluctuation of energy
3. dia of the crankshaft

Turning moment diagrams for Different types of engines:

1. turning moment diagram for a single-cylinder Double Acting steam engine:

turning moment on the crank shaft is given by

$$T = F_T \times r = F_p r \left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right] \text{ (or) } = F_p \times r \left(\frac{\sin(\theta+\phi)}{\cos\phi} \right)$$

F_T = crank-pin effort

F_p = piston effort

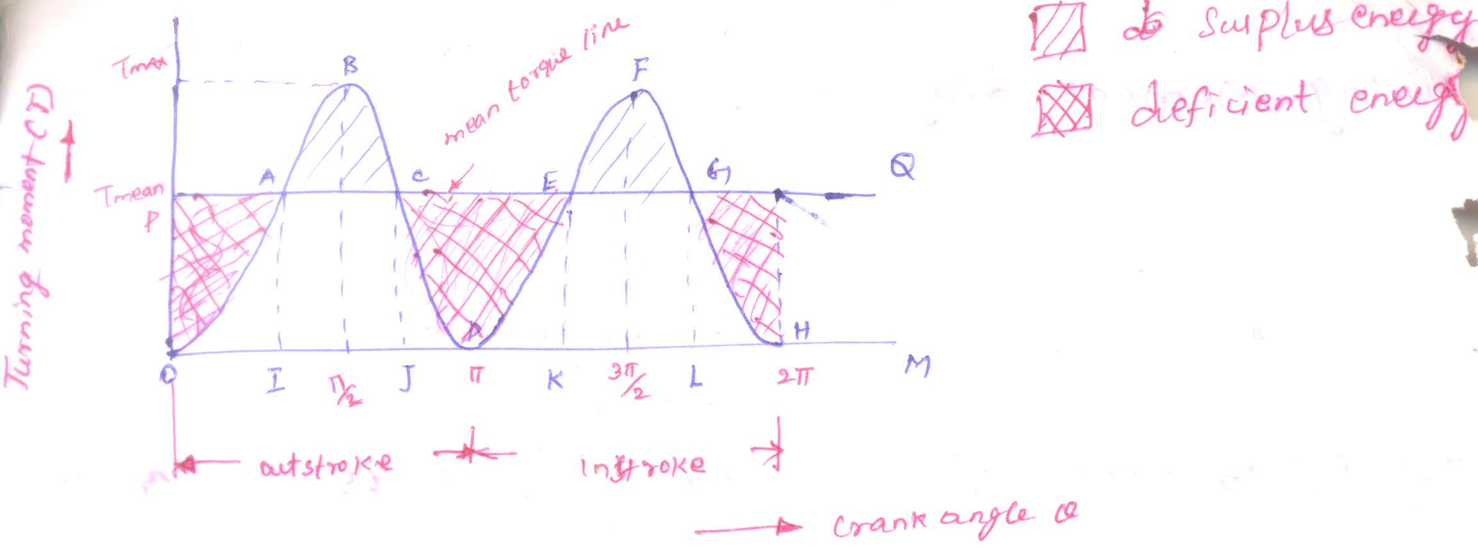
r = Radius of crank,

l = length of the connecting rod.

(obliquity ratio) $n = \frac{l}{r}$

θ = Angle turned by the crank from IDC.

above expression value of turning moment (T) varies with the variation of the crank rotation Angle (θ). is called turning moment diagram.



turning moment diagram for the outstroke of the piston ($\theta = 0°$ to $180°$) is shown by curve OABCD and the instroke of the piston is $\theta = 180°$ to $360°$ is DEFGH

$$T_{mean} = \frac{\text{Area } OABCDEFGH}{2\pi}$$

when the crank moves from O to I, energy required ~~of~~ OIAP while the energy produced is OAI. Therefore the deficiency of energy is OPA.

So the engine speed decreases during this Revolution.

When the crank move from I to J, the energy required & developed are the areas IACJ & IABCJ respectively.

Thus surplus excess energy is area ABC which increases the speed of the flywheel.

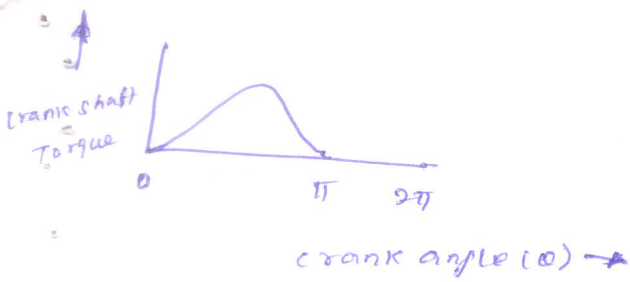
flywheel stores this excess energy & delivers it back to the engine.

crank move from J to K ; more energy is taken from the engine so energy will be loss by the area CDE

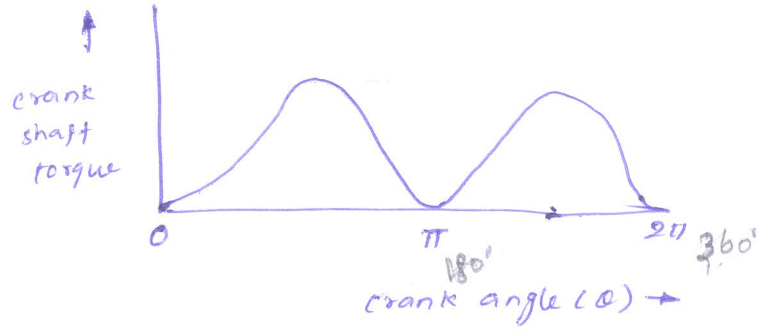
to overcome the loss flywheel give the energy to the engine speed decreases during this period.

turning moment diagram of Common Engines

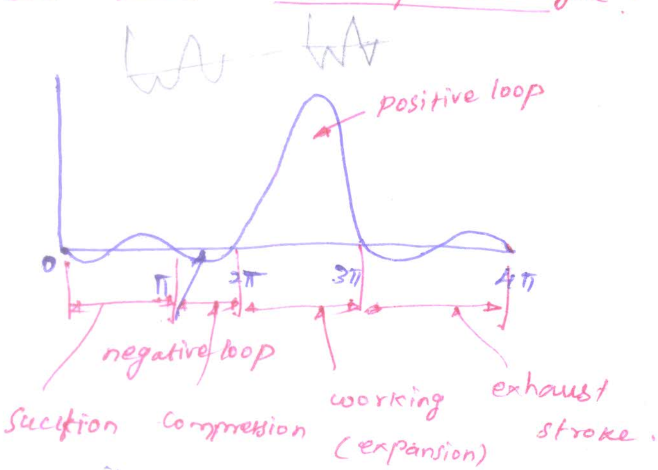
Single acting steam engine



Double acting steam engine

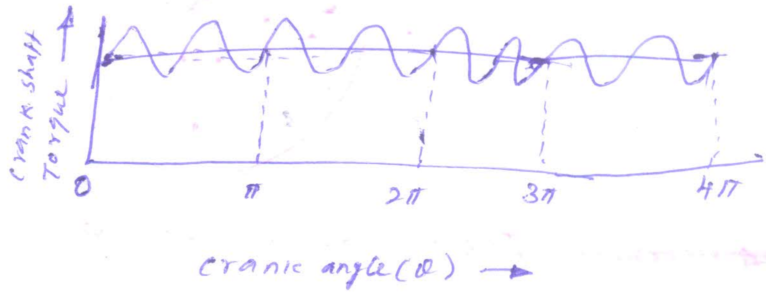


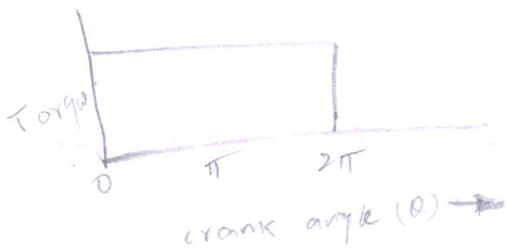
Four stroke single cylinder engine:



Power is obtained during working (or) expansion stroke only.
suction, comp, exhaust the energy is absorbed.

8 cylinder, 4 stroke engine (multicylinder Engine)





Turbine, electric motor (or) pelton wheel

Flywheel

A flywheel serves as a mechanical reservoir for storing mechanical energy.

Its function to store the energy during the period when the supply of energy is more than the requirement and to away same

(x) flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.

Appl

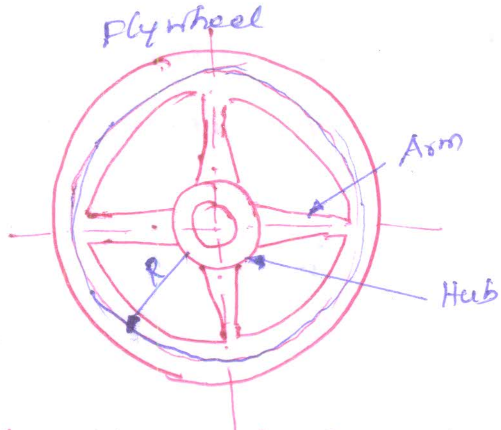
1. Flywheels are provided in engines & fabricating machines such as presses, shearing m/c, riveting m/c, Punching m/c, steel rollers.

for example:

IC engine the energy is developed during expantion stroke (or) power stroke. during high load (or) other suction, comp, exhaust strokes no energy developed.

So at the time flywheel give the energy to the the crank shaft.

reduces the fluctuation in the speed of engine.



1. when flywheel absorb the energy speed 11 increases
2. Flywheel releases the energy the speed of flywheel decreases.

~~maximum~~ Fluctuation of energy:

~~fluctuatio~~ The variation of energy above and blow the mean resisting torque line is called fluctuation of energy.

Maximum fluctuation of energy: $\Delta E \rightarrow$

The difference b/w maxi & min energies is known as maximum fluctuation of energy.

$$\boxed{\Delta E = \text{maximum energy} - \text{minimum energy}}$$

$\Delta E = I\omega^2 C_s$

mk^2

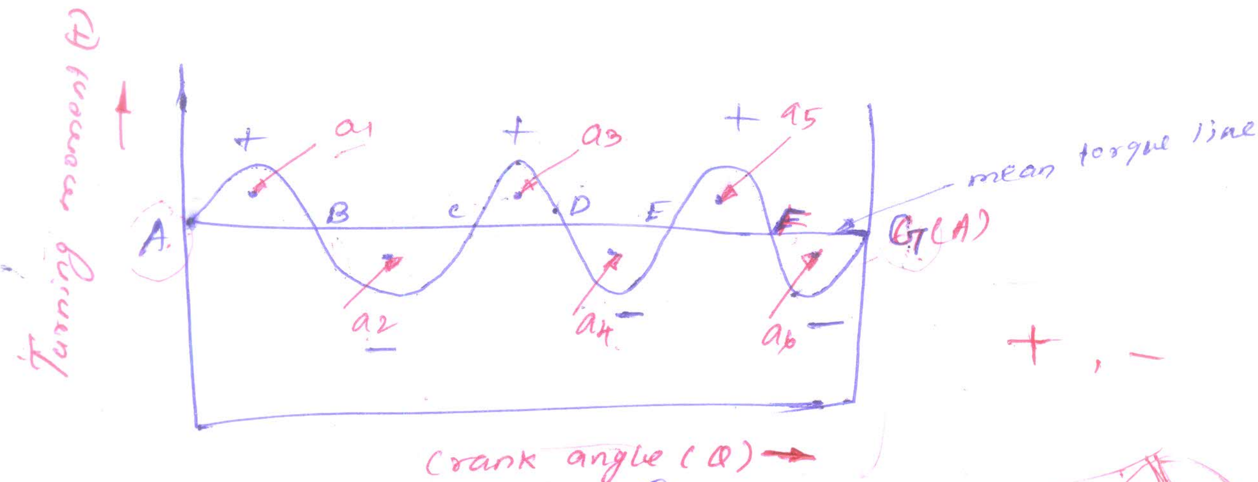
co-efficient of fluctuation of speed

Determination of Maximum Fluctuation of Energy:

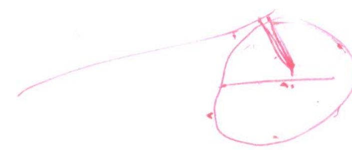
Fig represents the turning moment diagram of a multi-cylinder engine

The horizontal line AG represents the mean torque line,

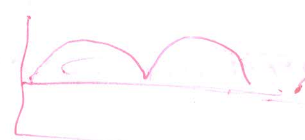
let a_1, a_3, a_5 be a areas above the mean torque a_2, a_4, a_6 Below the mean torque.



+ , −



⊗ Energy on the flywheel @ A = E



⊗. Crank position	Flywheel energy
A	E
B	$E+a_1$
C	$E+a_1-a_2$
D	$E+a_1-a_2+a_3$
E	$E+a_1-a_2+a_3-a_4$
F	$E+a_1-a_2+a_3-a_4+a_5$
G (or) A	$E+a_1-a_2+a_3-a_4+a_5-a_6$

$E+0$

Coefficient of Fluctuation of energy (C_E)

The ratio of the Maxi (ΔE)

ΔE = mix − mi

$= (\cancel{E}+a_1) - \cancel{E} + -$

$$\boxed{C_E = \frac{\text{maxi fluctuation of energy } (\Delta E)}{\text{workdone per cycle}}}$$

Relations to Find workdone/cycle:

1. workdone per cycle $= T_{mean} \times \theta$

$T = I\alpha$

$T = mk^2\alpha$

$T_{mean} \Rightarrow$ mean resisting torque in N-m

$\theta \Rightarrow$ Angle turned in one revolution in radians.

$\theta = 2\pi$ in case two-stroke IC engines

$\theta = 4\pi$ in case 4-stroke IC engines

$P = \frac{2\pi N T_{mean}}{60} \Rightarrow T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$ $\left(\omega = \frac{2\pi N}{60}\right)$

P = power transmitted in watts.

N = speed in rpm

ω = Angular velocity $= \frac{2\pi N}{60}$

2. workdone / cycle $= \frac{P \times 60}{n}$ $\Rightarrow$ $n \Rightarrow$ no. of working stroke / min

(single/double acting) $\rightarrow$ = N in case of 2-stroke IC engines

= N/2 in " " 4-stroke IC " steam engine

Maximum Fluctuation of speed (ΔS)

The diff b/w maxi & min speed during ~~the~~ a cycle is called

$$\boxed{\Delta S = \text{Maximum speed} - \text{Minimum speed}} = N_1 - N_2$$

Coefficient of fluctuation of speed (C_s)

$$\boxed{C_s = \frac{\text{Maximum fluctuation of speed}}{\text{mean speed}}} = \frac{N_1 - N_2}{N}$$

N_1 & N_2 = maxi & mini speeds during the cycle.

mean speed during the cycle $N = \frac{N_1 + N_2}{2}$

$$C_s = \frac{N_1 - N_2}{N} \Rightarrow \frac{N_1 - N_2}{\left[\frac{N_1 + N_2}{2}\right]}$$

(Value of Cs depends upon the nature of appli to the flywheel)

$$\boxed{C_s = \frac{2(N_1 - N_2)}{N_1 + N_2}} \leftarrow \text{in terms of speeds}$$

$$\boxed{C_s = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}} \leftarrow \text{angular speeds}$$

$$\boxed{C_s = \frac{2(V_1 - V_2)}{V_1 + V_2}} \leftarrow \text{linear speeds}$$

Coefficient of steadiness (m)

The reciprocal of the coefficient of fluctuation of speed is known as coeffic of steadiness. denoted by 'm'

$$\boxed{m = \frac{1}{C_s} = \frac{N}{N_1 - N_2}}$$

Energy stored in a flywheel:

When the flywheel absorbs the energy the speed of flywheel increases.

When the flywheel releases the energy, the speed of flywheel decreases.

diagram

m = Mass of the flywheel in kg.

K = Radius of gyration of the flywheel in m.

$I = mK^2$ = mass moment of inertia of the flywheel

N_1 & N_2 = max & mini speeds during the cycle in rpm.

ω_1 & ω_2 = Max & mini angular speeds " " rad/s

~~= N_1 N_2~~

$N = \frac{N_1+N_2}{2}$ = $\boxed{\text{mean speed}}$ during the cycle in rpm

$\omega = \frac{\omega_1+\omega_2}{2}$ = Mean angular speed during the cycle in rad/s

$C_s = (N_1-N_2)/N$ = coefficient of fluctuation of speed.

wkt Energy stored in a flywheel, $E = \frac{1}{2}I\omega^2$

Maxi " " " " $E_{max} = \frac{1}{2}I\omega_1^2$

mini " " $E_{min} = \frac{1}{2}I\omega_2^2$

$\therefore$ maxi fluctuation of energy is given by ΔE

ΔE = max energy (E_1) − min energy (E_2)

$E_1 = \frac{1}{2}I\omega_1^2$

$E_2 = \frac{1}{2}I\omega_2^2$

$$= \frac{1}{2}I\omega_1^2 - \frac{1}{2}I\omega_2^2$$

$$\boxed{\Delta E = \frac{1}{2}I(\omega_1^2-\omega_2^2)} \quad (1)$$

$$= \frac{1}{2}I(\omega_1+\omega_2)(\omega_1-\omega_2)$$

$\frac{1}{2}I(2\omega)(\omega_1-\omega_2)$

$I\omega(\omega_1+\omega_2)$

$\because \omega = \frac{\omega_1+\omega_2}{2}$

$$\boxed{\Delta E = I\omega(\omega_1-\omega_2)} \quad (2)$$

Energy stored in a flywheel:

When the flywheel absorbs the energy the speed of flywheel increases.

When the flywheel releases the energy, the speed of flywheel decreases.

diagram

m = Mass of the flywheel in kg.

K = Radius of gyration of the flywheel in m

$I = mK^2$ = mass moment of inertia of the flywheel

N_1 & N_2 = max & mini speeds during the cycle in rpm.

ω_1 & ω_2 = Max & mini angular speeds " " rad/s

~~N = N1+N2~~

$N = \frac{N_1+N_2}{2}$ = $\boxed{\text{mean speed}}$ during the cycle in rpm

$\omega = \frac{\omega_1+\omega_2}{2}$ = Mean angular speed during the cycle in rad/s

$C_s = (N_1-N_2)/N$ = coefficient of fluctuation of speed.

wkt Energy stored in a flywheel, $E = \frac{1}{2}I\omega^2$

Maxi " " " " $E_{max} = \frac{1}{2}I\omega_1^2$

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ΔE = max energy (E_1) − min energy (E_2)

$E_1 = \frac{1}{2}I\omega_1^2$

$E_2 = \frac{1}{2} I\omega_2^2$

$= \frac{1}{2}I\omega_1^2 - \frac{1}{2}I\omega_2^2$

$\frac{1}{2}I(2\omega)(\omega_1-\omega_2)$

$I\omega(\omega_1+\omega_2)$

$$\boxed{\Delta E = \frac{1}{2}I(\omega_1^2-\omega_2^2)} \quad \text{— (1)}$$

$= \frac{1}{2}I(\omega_1+\omega_2)(\omega_1-\omega_2)$

$\because \omega = \frac{\omega_1+\omega_2}{2}$

$$\boxed{\Delta E = I\omega(\omega_1-\omega_2)} \quad \text{— (2)}$$

multiplying & dividing eq (2) by 'ω' we get

$$\cancel{\Delta E = I\omega^2 [}$$

$$\left(\therefore C_s = 2\frac{\omega_1-\omega_2}{\omega_1+\omega_2}\right)$$

$$\Delta E = I\omega(\omega_1-\omega_2) \times \frac{\omega}{\omega}$$

$$= \frac{I\omega^2(\omega_1-\omega_2)}{\omega}$$

$$\left(\therefore \omega = \frac{\omega_1+\omega_2}{2}\right)$$

$$= \frac{I\omega^2(\omega_1-\omega_2)}{\frac{\omega_1+\omega_2}{2}} = \frac{I\omega^2(\omega_1-\omega_2)\times 2}{\omega_1+\omega_2}$$

$$\boxed{\Delta E = I\omega^2 C_s} = \boxed{mk^2\omega^2 C_s} \qquad (\because I = mk^2)$$

$$\boxed{\Delta E = 2EC_s} \longrightarrow E = \frac{1}{2}I\omega^2$$

$$2E = I\omega^2$$

Thickness of rim is very small as compared to the dia of rim, the radius of gyration (K) is taken equal to the $\boxed{\text{mean radius}}$ of the rim (R)

R

$K = R$

$$\Delta E = mk^2\omega^2 C_s$$

$$\Delta E = mR^2\omega^2 C_s \qquad (\because V = R\omega)$$

$$\boxed{\Delta E = mV^2 C_s}$$

1. An engine flywheel has a mass of 6.5 tonnes and the radius of gyration is 2m. If the maximum & min speeds are 120 rpm. & 118 rpm respectively, find

(i) mean speeds of flywheel (N)

(ii) Coefficient of fluctuation of speed (C_s)

(iii) maximum fluctuation of energy (ΔE)

1 tonne = 1000 kg

given data:

m = 6.5 tonnes = 6.5 × 1000 = 6500 kg

K = 2 m

N_1 = 120 rpm

N_2 = 118 rpm

~~mass~~ ~~moment of~~

(i) mean speed of flywheel (N):

mean speed $N = \frac{N_1 + N_2}{2} = \frac{120 + 118}{2} = 119$ rpm

$\boxed{N = 119 \text{ rpm}}$

(ii) coefficient of fluctuation of speed (C_s): maxi $\frac{\text{fluctuation of speed}}{\text{mean speed}}$

$C_s = \frac{N_1 - N_2}{N} = \frac{120 - 118}{119}$ $\boxed{C_s = 0.0168}$

(iii) maximum fluctuation of energy (ΔE):

$\boxed{\Delta E = I \omega^2 C_s}$

$I = mk^2 = 6500 \times (2)^2 = \boxed{26000 \text{ kg-m}^2 = I}$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 119}{60} = \boxed{12.45 \text{ rad/s} = \omega}$

$\Delta E = I\omega^2 C_s = 26000 \times (12.45)^2 \times 0.0168$

$\Delta E = 67763$ N.m $= \frac{67763}{1000}$

$\boxed{\Delta E = 67.763 \text{ kN-m}}$

Formula,

Acceleration torque T_A

$T_A = T - T_{mean}$

H.W

2. The rim of an engine flywheel is of mass of 7 tonnes & mean radius of rim being 2 meters. If is found from crank-effort diagram that the fluctuation of energy is 90 kJ. If the mean speed is 120 rpm find the coefficient of fluctuation of speed and the limiting speeds of the flywheel.

given: m = 7 tonnes = 7000 kg, R = 2 m; ΔE = 90 kJ = 90 kN-m, N = 120 rpm

(i) coefficient of fluctuation of speed (C_s):

$\boxed{\text{N-m} = \text{J}}$

$\Delta E = I\omega^2 C_s = mk^2 \omega^2 C_s$

$C_s = 0.0203$ (or) 2.03%

$I = mk^2$

K = R

$\omega = \frac{2\pi N}{60}$

(ii) Limiting speeds i.e. max & mini speeds (N_1 & N_2)

coefficient of flue of speed (C_s):

$$C_s = \frac{N_1 - N_2}{N_0}$$ here consider $N_1 = N$

$$0.0203 = \frac{N_1 - N_2}{120} \Rightarrow N_1 - N_2 = 2.4426$$

$120 - N_2 = 2.4426$

$-N_2 = 2.4426 - 120$

$-N_2 = -117.55$

$\boxed{N_2 = 117.6}$ rpm

$$N = \frac{N_1 + N_2}{2}$$

$$120 = \frac{N_1 + N_2}{2}$$

$240 = N_1 + N_2 \Rightarrow 240 = 117.6 + N_1$

$N_1 + N_2$

~~$N_2 = 22.4$ rp~~

$\boxed{N_1 = 122.4 \text{ rpm}}$

$240 = N_1 + 117.6$

$240 - 117.6 = N_1$

$\boxed{N_1 = 123.8 \text{ rpm}}$

3. A horizontal steam engine 20 cm dia by 40 cm stroke, connecting rod 100 cm makes 160 rpm. The mass of the reciprocating parts is 50 kg. when the crank has turned through an angle of 30°, the steam pressure is 4.5 bar.

(a) calculate the turning moment on crank shaft

(b) If the mean resistive torque is 30 N-m and the mass of flywheel is 50 kg and the radius of gyration 70 cm, calculate the acceleration of the flywheel.

given data:

$D = 20\text{cm} = \frac{20}{100} = 0.2\text{ m}$

Stroke (L) $= 40\text{ cm} = r = \frac{40}{2} = 20\text{ cm} = 0.2\text{ m}$

$l = 100\text{ cm} = 1\text{ m}$

flywheel $m = 50$ kg

$N = 160$ rpm.

$m_R = 50$ kg

$k = 70\text{ cm} = 0.7\text{ m}$

$\theta = 30°$

$p = 4.5\text{ bar} = 4.5 \times 10^5\text{ N/m}^2$

$T_{mean} = 30\text{ N-m}$

To find:

Sol)

(a) Turning moment on the crankshaft (T):

$$T = F_P \times r \left(\frac{\sin \theta + \phi}{\cos \phi} \right)$$

load on the piston

inertia force on recips parts

(piston effort) $F_P = F_L - F_I$

$$p = \frac{F_L}{A} = F_L = p \times A \Rightarrow 4.5 \times 10^5 \times 0.0314 = \boxed{14137.2\ N}$$

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.2) = 0.0314\ m^2$$

Inertia force on the reciprocating parts.

$$\omega = \frac{2\pi N}{60} = \frac{2\pi (160)}{60} = 16.75\ r/s$$

$$F_I = m_R \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right) = 50(16.75)^2 \times 0.2 \left(\cos(30) + \frac{\cos 2(30)}{5} \right)$$

$$n = \frac{l}{r} = \frac{1}{0.2} = 5$$

$$\boxed{F_I = 2711.97\ N}$$

Piston effort (or) net force on the piston,

$$F_P = F_L - F_I = 14137.2 - 2711.97$$

$$\boxed{F_P = 14137.2\ N}$$

WKT

$$\sin\phi = \frac{\sin\theta}{n} \Rightarrow \frac{\sin 30}{5} = 0.1 \quad \phi = \sin^{-1}(0.1) = 5.74^\circ$$

$$\boxed{\phi = 5.74^\circ}$$

WKT turning moment on the crankshaft.

$$T = F_P \times r \frac{\sin(\theta + \phi)}{\cos\phi}$$

$$= 14137.2(0.2) \frac{\sin(30 + 5.74^\circ)}{\cos(5.74^\circ)}$$

$$\boxed{T = 1659.85\ N\text{-}m}$$

Acceleration of the flywheel (α)

Acceleration of flywheel takes place when the turning moment (T) is more than the resisting torque (T_{mean}) so

$$\text{Accelerating torque, } T_A = T - T_{mean}$$

$$\text{Acc torque } T_A = I\alpha = (mk^2)\alpha \qquad I = mk^2$$

$$T_A = T - T_{mean} = 1659.85 - 30$$

$$\boxed{T_A = 1629.85 \text{ N-m}}$$

$$T_A = I\alpha = m(k^2)\alpha$$

$$1629.85 = 50 \times (0.7)^2 \times \alpha$$

$$\alpha = \frac{1629.85}{50 \times (0.7)^2} \qquad \boxed{\alpha = 66.52 \text{ rad/s}^2}$$

H.W

4. A vertical double-acting steam engine develops 75 KW at 250 rpm. The max fluctuation of energy is 30% of the workdone per stroke. The max & min speeds are not to vary more than $\pm 1\%$ on either side of the mean speed. Find the mass of the flywheel required if the radius of gyration is 0.6 meters.

give data:

$P = 75$ KW ; $N = 250$ rpm, $\Delta E = 30\%$ of workdone / stroke

$C_s = \pm 1\% = 2\% = 0.02$; $k = 0.6$ m

this formul not use
Q is not given so

$\cancel{T_{mean}} \quad P = \frac{2\pi N T_{mean}}{60}$

Sol)

$$\text{workdone / cycle} = \frac{P \times 60}{N} \text{ (or) } \frac{P \times 60}{n} \quad (N = n \text{ double acting eng}) \text{ (or) } T_{mean} \times Q$$

$$\Rightarrow 18000 \text{ N-m}$$

double / single

If steam engine means should consider N rpm = n (no of working stroke/m)

$$\Delta E = 30\% \text{ workdone / cycle}$$

$$= \frac{30}{100} \times 18000 = 5400 \text{ N-m}$$

coefficient of fluct of speed

$C_s = \frac{\text{max fluct of speed}}{\text{mean speed}}$

$C_s = \frac{N_1 - N_2}{N}$

$$\Delta E = I\omega^2 C_s = mk^2\omega^2 C_s$$

$$m = \frac{\Delta E}{k^2 \omega^2 C_s} \qquad \boxed{m = 1094.26 \text{ kg}}$$

H.W

5. The radius of gyration of a flywheel is 1meter and the fluctuation of speed is not exceed 1% of the mean speed of the flywheel. if the mass of the flywheel is 3340 kg & the steam engine develops 150 KW @ 135 rpm, then find.

(1) max fluctuation of energy &

(2) Coefficient of fluctuation of energy

X: if giving steam engine should consider N rpm = n (no of work stroke/m)

Given data:

$k = 1m$, $C_s = 1\% = 0.01$, $m = 3340 kg$, $P = 150 kW$, $N = 135 rpm$

(i) Maxi fluctuation of energy (ΔE):

$$\Delta E = I \omega^2 C_s = 6675.3 \ N\text{-}m$$

(ii) coefficient of energy (C_E)

$$C_E = \frac{\text{Max fluctuation of energy } (\Delta E)}{\text{workdone / cycle}}$$

$$w/\text{cycle} = \frac{P \times 60}{n} = 66,666.7 \ N\text{-}m$$

$n = 135$

$$C_E = \frac{6675.3}{66666.7} = 0.1$$

6. The turning moment diagram for a petrol engine is drawn to a vertical scale of 1mm to 6Nm & horizontal scale of 1mm to 1°. the turning moment repeats itself after every half revolution of the engine. The areas above & below the mean torque line are 305, 710, 50, 350, 980, & 275 mm^2. mass of rotating parts is 40 kg at a radius of gyration of 140 mm. Calculate the coefficient of fluctuation of speed if the mean speed is 1500 rpm

Given data: $m = 40$ kg

$K = 140$ mm

$N = 1500$ rpm

Sol) ~~Mean torque~~

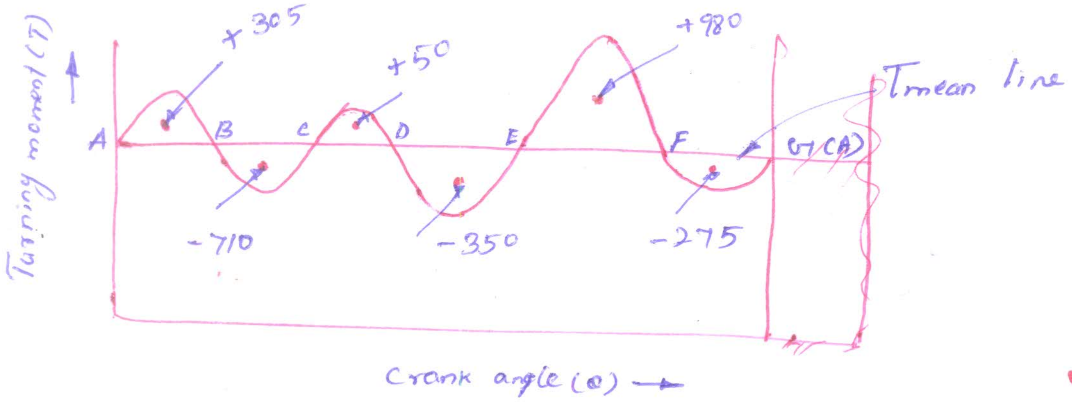
given that,:

Turning moment, 1 mm = 6 N-m &

Crank angle, 1 mm = 1° → ($\pi/180$ is to convert to radian)

$\therefore$ 1 mm² on turning moment diagram $= 6 \times (1^\circ \times \frac{\pi}{180}) = 0.1047$ N-m ≈ 0.105 N-m

The turning moment diagram for the given data.



$\omega = \frac{2\pi N}{60}$ rad/sec

All areas in mm²

Let the total energy @ A = E.

Point (or) crank position	flywheel Energy (mm²)		
A	E		E
B	E + 305		E + 305 (max energy)
C	E + 305 − 710	= ~~E − 405~~	E + 305 − 710 = E − 405
D	E + 305 − 710 + 50	=	E − 405 + 50 = E − 355
E	E + 305 − 710 + 50 − 350	=	E − 355 − 350 = E − 705 (min Eng)
F	E + 305 − 710 + 50 − 350 + 980	=	E − 705 + 980 = E + 275
G	E + 305 − 710 + 50 − 350 + 980 − 275	=	E + 275 − 275 = E + 0 = E = energy @ A

correct

wkt,

$$\Delta E = I\omega^2 C_s = mk^2\omega^2 C_s$$

$\therefore \Delta E$ = Max energy − min energy = Energy @ B − Energy @ E

max fluct of Energy

$$\Delta E = (E+305)-(E-705)$$

$$= 1010 \text{ mm}^2$$

$$\therefore = 1010 \times 0.105 = \boxed{106.05 \text{ N-m} = \Delta E}$$

$(1 \text{ mm}^2 = 0.105 \text{ Nm})$

$$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 1500}{60}$$

$$\boxed{\omega = 157.08 \text{ r/s}}$$

$$\Delta E = I\omega^2 C_s = mk^2\omega^2 C_s$$

$$106.05 = 40\times(140)^2\times(157.08)^2\times C_s$$

$$C_s = \frac{106.05}{40\times(140)^2\times(157.08)^2}$$

(or) $C_s = \frac{\text{maxi flu of speed}}{\text{mean speed}} = \frac{106.05}{1500}$

(if asking mass)

Coefficient of fluctuat of speed $\boxed{C_s = 0.0058 \text{ (or) } 0.548\,\%}$

$$= \frac{106.05}{10816\times 40\times} \quad \frac{}{1.067}$$

7. The torque delivered by a two-stroke engine is represented by $T = (1000 + 300\sin 2\theta - 500\cos 2\theta)$ N-m, where θ is the angle turned by the crank from the inner-dead centre. The engine speed is 250 rpm. The mass of the flywheel is 400 kg and radius of gyration 400 mm. Determine.

(i) the power developed

(ii) The total percentage fluctuation of speed, $T = T_2 - T_1$

(iii) the angular acceleration of flywheel when the crank has rotated through an angle of 60° from the inner dead centre

(iv) the maximum angular acceleration & retardation of the flywheel. (AP/may 2003, AP/may 2007)

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Given data:

$T = (1000 + 300 \sin 2\theta - 500 \cos 2\theta)$ N-m;

$N = 250$ rpm

$m = 400$ kg

$K = 400$ mm $= \frac{400}{1000} = 0.4$ m

To Find:

(i) Power developed (P)

(ii) total " (C_s) %

(iii) α @ $\theta = 60°$

(iv) max α

$\theta = 360° \Rightarrow 2\pi$

$2\theta = \frac{360°}{2} = 180° = \pi$

$3\theta = \frac{360}{3} = 120° \; \frac{2\pi}{3}$.

$180° \times 2 = 360$

360, 180, 180, 120, 120, 120, 360

Sol

(i) Power developed by the engine (P)

since the given torque equation is a function of 2θ, the cycle of operation will be repeated after every 180° (or) π rad of the crank rotation.

Power developed $P = \frac{2\pi N T_{mean}}{60} =$ ~~$T_{mean} = \frac{P}{\omega}$~~ $\quad \omega = \frac{2\pi N}{60}$

$P = T_{mean} \times \omega \qquad = \frac{2\pi \times 250}{60} = 26.18$ r/s

will not use in this problem

$\frac{n \times 60}{n}$

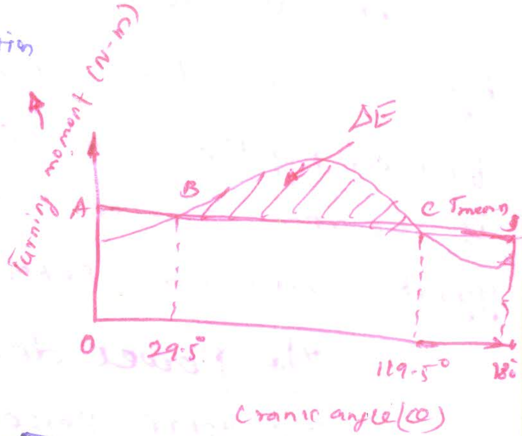
workdone / cycle = $T_{mean} \times \theta$

$T_{mean} = \frac{\text{workdone / cycle}}{\text{crank angle per revolution}}$

workdone / cycle $= \int_0^{\pi} T \cdot d\theta$

$\int \sin\theta = -\cos\theta$

$\int \cos\theta = \sin\theta$



$= \int_0^{\pi} (1000 + 300 \sin 2\theta - 500 \cos 2\theta) \cdot d\theta$

$= \left[1000(\theta) + 300\left(\frac{-\cos 2\theta}{2}\right) - 500\left(\frac{\sin 2\theta}{2}\right) \right]_0^{\pi}$

$\sin 2\pi = 0$

$= \left[\left(1000\pi + 300\left(\frac{-\cos 2\pi}{2}\right) - 500\left(\frac{\sin 2\pi}{2}\right)\right)\right] - \left[1000(0) + 300\left(\frac{-\cos 2(0)}{2}\right) - 500\left(\frac{\sin 2(0)}{2}\right)\right]$

$= [1000\pi - 150 - 0] - [0 - 150 - 0]$

$= 1000\pi$ N-m

∴ Mean resisting Torque, $T_{mean} = \dfrac{\text{workdone per cycle}}{\text{crank angle per Revolution}} = \dfrac{1000\pi}{\pi}$

two stroke means

$$\boxed{T_{mean} = 1000 \text{ N-m}}$$

Power developed, $P = T_{mean} \times \omega$

$= 1000 \times 26.18$

$= 26180 \text{ W}$

$= 26180/1000 = 26.18 \text{ kW}$

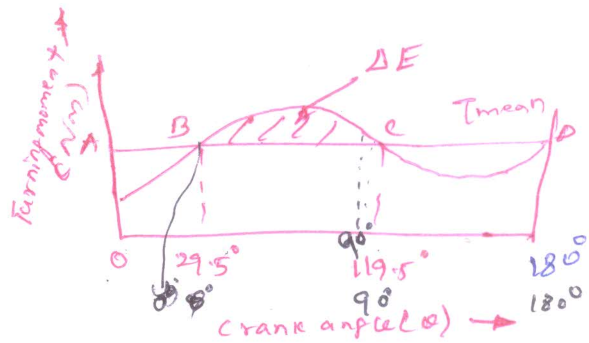
$P = \dfrac{2\pi N T_{mean}}{60}$

$$\boxed{P = 26.18 \text{ kW}}$$

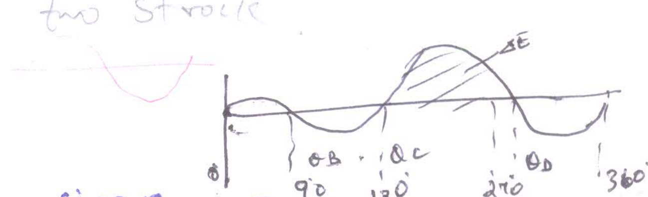
(ii) Total percentage fluctuation of speed:

here the cycle of opeation is repeated in every π revolution, shown in fig.

since torque exerted at point B & C is equal to mean torque on the flywheel ∴



two strocke



$T = T_{mean}$

$1000 + 300\sin 2\theta - 500\cos 2\theta = 1000$

$300\sin 2\theta - 500\cos 2\theta = 1000 - 1000$

$300\sin 2\theta - 500\cos 2\theta = 0$

$300\sin 2\theta = 500\cos 2\theta$

$\sin 2\theta = \dfrac{500}{300}\cos 2\theta$

$\sin 2\theta = \dfrac{5}{3}\cos 2\theta \Rightarrow \dfrac{\sin 2\theta}{\cos 2\theta} = 5/3$

$\tan 2\theta = 5/3$

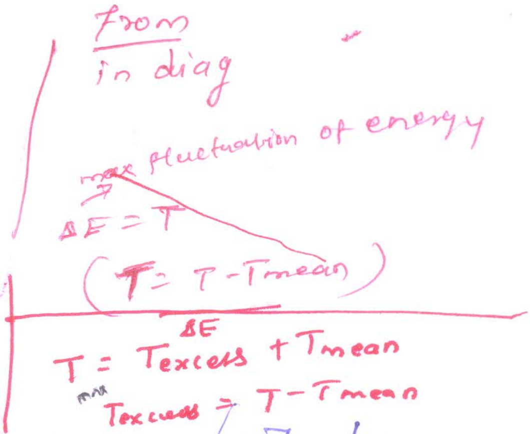
$59 + 180$

$2\theta = 59^\circ$ (or) $2\theta = 239^\circ$

$29.5^\circ + 90^\circ = 119.5$

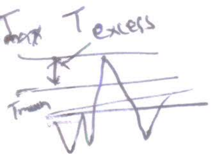
$$\boxed{\begin{aligned}\theta_B &= 29.5^\circ \\ \theta_C &= 119.5^\circ\end{aligned}}$$

$\Delta E = I\omega^2 C_s = mk^2\omega^2 C_s$



maximum fluctuation of energy:

$$\Delta E = \int_{29.5^\circ}^{119.5^\circ} \underbrace{(T - T_{mean})}_{T_{excess}} \cdot d\theta$$



$$= \int_{29.5^\circ}^{119.5^\circ} [(\cancel{1000} + 300\sin 2\theta - 500\cos 2\theta) - \cancel{1000}]\, d\phi$$

$$= \left[300\left(-\frac{\cos 2\theta}{2}\right) - 500\left(\frac{\sin 2\theta}{2}\right)\right]_{29.5^\circ}^{119.5^\circ}$$

$$= \left[300\left(-\frac{\cos 2(119.5)}{2}\right) - 500\left(\frac{\sin 2(119.5)}{2}\right)\right] - \left[300\left(\frac{-\cos 2(29.5)}{2}\right) - 500\left(\frac{\sin 2(29.5)}{2}\right)\right]$$

$$= [77.25 + 214.29] - [-77.25 - 214.29]$$

$$\boxed{\Delta E = 583.08 \text{ N-m}}$$

wkt $\quad \Delta E = I\omega^2 C_s = mk^2\omega^2 C_s$

T_{mean}

$$583.08 = 400 \times (0.4)^2 \times (26.18)^2 \times C_s$$

$$\boxed{C_s = 0.01329 \text{ (or) } 1.329\%}$$

(iii) Angular acceleration of flywheel when $\theta = 60^\circ$

The angular acceleration of flywheel is produced by the the excess torque over the mean torque. ∴ (T_{mean})

T_{excess}

$T_{ex} = (T - T_{mean})$ $\qquad$ $T = T_{ex} + T_{mean}$ $\qquad$ $T_A = I\alpha$ $\qquad$ $T_{ex} = T_{max} - T_{mean}$

$I\alpha = (T - T_{mean})$ $\qquad$ $I = mk^2$

$\alpha = \left(\dfrac{T - T_{mean}}{I}\right)$ $\qquad$ 509.

$T = T_{ex} + T_{mean}$ $\quad$ $T_{max} - T_{mean}$

$$\alpha = \frac{(1000 + 300\sin 2\theta - 500\cos 2\theta) - 1000}{400 \times (0.4)^2}$$

$$@\ \theta = 60^\circ \Big/ = \frac{259.8 + 250}{64} \qquad \boxed{\alpha = 7.965\ r/s^2}$$

(iv) maxi acceleration & retardation (or) min acceleration of the flywheel :

acceleration in the flywheel is produced by excess torque over the mean torque

rate of charge of velocity

$$\frac{d}{d\theta}(T - T_{mean}) = 0$$

$$\frac{d}{dx} \sin 2\theta = \cos 2\theta \times 2 \qquad \cos 2\theta = -\sin 2\theta \times 2$$

$\alpha p = d v p \times \omega_1$

angular velocity

$$\frac{d}{d\theta}\left[(1000 + 300\sin 2\theta - 500\cos 2\theta) - 1000\right] = 0$$

$\cos 2\theta = 2\cos^2\theta - 1$

$$300\cos 2\theta \times 2 - 500(-\sin 2\theta \times 2) = 0$$

$$600\cos 2\theta + 1000\sin 2\theta = 0 \qquad \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$600\cos 2\theta = -1000\sin 2\theta \qquad \frac{600}{-1000} = -0.6$$

$$\cos 2\theta = \cancel{-1.66}\ \sin 2\theta$$

$$\frac{600}{-1000} = \frac{\sin 2\theta}{\cos 2\theta}$$

$$\tan 2\theta = -0.66 = 2\theta = -30.96 + 180$$

$$0.66 = \tan 2\theta$$

$$2\theta = \tan^{-1}(0.66)$$

$$2\theta = 149.04^\circ \ \& \ 329.04^\circ \leftarrow \begin{array}{c} 149.04 + \\ 180 \end{array}$$

When $2\theta = 149.04^\circ$, $T_{1_{ex}} = T_{max} - T_{mean}$

$T_{max} = T_{ex} + T_{mean}$

$1200\cos 2\theta \cancel{\times 2} - 1)^*$

$$= 1000 + 300\sin 2\theta - 500\cos 2\theta - 1000$$

$$= 300\sin 2\theta - 500\cos 2\theta$$

$$= 300\sin(149.04) - 500\cos(149.04)$$

$$T_1 = 583.1\ N\text{-}m$$

When $2\theta = 329.04^\circ$, $T_1 = T - T_{mean}$

$$= 300\sin(329.04) - 500(\cos(329.04))$$

$$T_1 = -583.1\ N\text{-}m$$

As values of $T - T_{mean}$ @ max & min torque T are same ∴ the max acceleration is equal to max retardation.

$T - T_{mean} = T \Rightarrow I\alpha$ here $T = I\alpha$ / $I = mk^2$

$T - T_{mean} = mk^2\alpha$

(min acce = maxi retardation)

$583.1 = 400 \times (0.4)^2 \times \alpha$

max acceleration $\boxed{\alpha = 9.1109 \text{ rad/s}^2}$

if T value is diff means of

$T_{max} = I\alpha \Rightarrow \alpha = \frac{T_{max}}{I \leftarrow mk^2}$

$T_{min} = I\alpha \Rightarrow \alpha = \frac{T_{min}}{I \rightarrow mk^2}$

HW pg 3.29

8. The torque exerted on the crank shaft of a two-stroke engine is given by the equation: $T(N\text{-}m) = 14{,}500 + 2300 \sin 2\theta - 1900 \cos 2\theta$ where θ is the crank angle displacement from the inner dead centre. Assuming the resisting torque to be constant,

Determine:

1. The power of the engine when the speed is 150 rpm.
2. The moment of inertia of the flywheel if the speed variation is not to exceed $\pm$ ~~0.5~~ 1% of the mean speed. &
2. The angular acceleration of the flywheel when the crank has turned through 30° from the IDC,

(same procedure like last some problem)

give

$C_s = \pm 0.5\% = 1\% = 0.01$

Flywheels of Punching Presses

Formulas

Area of rim $= A = b \times t$ | area of plate $A = \pi d \times t_1$

$$\text{power} = \frac{\text{Energy required / min}}{60 \times \eta_m}$$

Energy required / min = Energy / stroke × no. of working strokes / min

Energy required / stroke = Average shear force × thickness of plate (t)

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} \text{ (or) } \frac{\text{force}(F_s)}{\text{Area } A_0} \qquad = \frac{1}{2} \times F_s \times t$$

force = Area × stress

$\Delta E = I \omega^2 C_s$ $\qquad I = mk^2$ $\qquad k = R$

$\Delta E = m R^2 C_s \omega^2$

(bob kuru)

1. A punching machine makes 25 working strokes per minute and is capable of punching 25 mm diameter holes in 18 mm thick steel plates having an ultimate shear strength 300 MPa. The punching operation takes place during 1/10th of a revolution of the crankshaft.

Estimate the power needed for the driving motor, assuming a mechanical efficiency of 95%. Determine suitable dimensions for the rim cross section of the flywheel, having width equal to twice thickness. The flywheel is to revolve @ 9 times the speed of the crankshaft. The permissible coefficient of fluctuation of speed is 0.1.

The flywheel is to be made of cast iron having a working stress (tensile) of 6 MPa & density of 7250 kg/m². The diameter of the flywheel must not exceed 1.4 m owing to space restrictions. The hub & the spokes may be assumed to provide 5% of the rotational intertia of the wheel.

Given data :

$n = 25$; $d_1 = 25\ mm = 0.025\ m$

$t_1 = 18\ mm = 0.018\ m$

ultima shear ~~stress~~ strength } $\tau_u = 300\ MPa = 300 \times 10^6\ N/m^2$

$\eta_m = 95\% = 0.95$; $C_s = 0.1$

$\sigma = 6\ MPa = 6 \times 10^6\ N/m^2$

$\rho = 7250\ kg/m^3$.

$D = 1.4\ m$ (or) $R = 0.7\ m$

sol

Power needed for the driving motor :

wkt power needed for the driving motor =

$$= \frac{\text{Energy required / min}}{60 \times \eta_m}$$

Energy required / min = Energy / stroke × no. of working strokes / min

Energy required / stroke = Avg shear force × thickness of plate

$= \frac{1}{2} \times F_s \times t_1$

$F_s = A_s \times \tau_u$

$A_s = \pi d_1 \times t_1 = \pi \times 0.025 \times 0.018$ $\boxed{= 1414 \times 10^{-6}\ m^2}$

$F_s = 1414 \times 10^{-6} \times 300 \times 10^6$

$\boxed{F_s = 424200\ N}$

energy required / stroke = $\frac{1}{2} \times F_s \times t_1 \Rightarrow \frac{1}{2} \times 424200 \times 0.018$

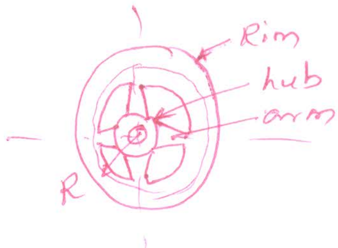
$\boxed{= 3817.8\ N\text{-}m}$

~~Power~~ energy required / min = 3817.8×25

$\boxed{= 95450\ N\text{-}m}$

Power = $\frac{95450}{60 \times 0.95} = 1675\ W \Rightarrow \boxed{1.675\ kW \Rightarrow P}$

Dimensions of the rim cross - section



$t = ?$ & $b = ? \rightarrow$ width of rim in metres

$b = 2t$

$\therefore$ cross - sectional area of rim

$$A = b \times t = 2t \times t = 2t^2$$

w.k.t mass of the flywheel (m)

$$m = \pi D \times A \times \rho$$

$$A = b \times t \Rightarrow 2t^2 \leftarrow (\because b = 2t)$$

$m =$ volume $\times$ density

$m = \pi D A \times \rho$

since the punching opeeation take place dueing $1/10^{th}$ of a revolution of a crank shaft of the crankshaft, $\therefore$ dueing $9/10^{th}$ of the revolution the energy is stored in the flywheel.

$\therefore$ maximum fluctuation of Energy

$$\Delta E = 9/10 \times \text{Energy / stroke} = 9/10 \times 3817.8 = 3436 \text{ N-m}$$

since the ~~flywheel~~ hub & the spoke provide 5% of the rotational inertia of the wheel. $\therefore$ of maximum fluctuation of energy provided by the flywheel by the rim will be 95%.

$$\Delta E_{rim} = 0.95 \times \Delta E = 0.95 \times 3436 = 3264 \text{ N-m}$$

since the flywheel is to revolve @ 9 times the speed of the crankshaft & there are 25 working strokes / minute, $\therefore$ mean speed of the flywheel.

$$N = 9 \times 25 = 225 \text{ rpm}$$

& mean angular speed,

$$\omega = \frac{2\pi \times 225}{60} = 23.56 \text{ rad/s}$$

w.k.t, max fluctuation of energy (ΔE_{rim})

$I = mk^2$

$k = R$

$$(\Delta E_{rim}) = mR^2 \omega^2 C_s$$

$3264 = \cancel{\quad} m \times (0.7)^2 \times (28.56)^2 \times 0.1$

$3264 = 27.2 m$

$m = 120 \text{ kg}$

mass of the fly wheel (m)

$m = \text{volume} \times \text{Density}$

$m = (\pi D A) \times \rho$

$A = \frac{m}{\pi D \rho}$

$120 = \pi D \times A \times \rho$

$120 = \pi \times 1.4 \times 2t^2 \times 7250 = 63782 t^2$

$t^2 = 120/63782$

$t = 0.044 m = 44 mm$

$\boxed{t = 44 mm \quad b = 88 mm}$

$b = 2t = 2 \times 44 = 88 mm$

3.44

A Punching press is required to punch 30mm diameter holes in a plate of 20mm thickness at the rate of 20 holes/minute. It requires 6 N-m of energy/mm² of sheared area. If punching takes place in 1/10th of a second & the speed of the flywheel varies from 160 to 140 rpm. determine the mass of the flywheel having radius of gyration 1m.

given data:

$d = 30 mm = 0.03 m$;

$t = 20 mm = 0.02 m$

$N_1 = 160 rpm$

$N_2 = 140 rpm$

$K = 1m$

$(\Delta E = m k^2 \omega^2 C_s$ not use this formula for this probl)

Sol

Angular velocity $\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 160}{60} = 16.75$ r/s

$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 140}{60} = 14.66$ r/s

31-12-1723 75-27 10C

max fluc of Energy

$\Delta E = E_1 - E_2$

$E_1 = \frac{1}{2} I\omega_1^2$, $E_2 = \frac{1}{2} I\omega_2^2$

$\Delta E = E_1 - E_2 = \frac{1}{2} I(\omega_1^2 - \omega_2^2)$

$I = mk^2$

$\Delta E = E_1 - E_2 = \frac{1}{2} mk^2 (\omega_1^2 - \omega_2^2)$

Energy required to punch a hole E_1 = sheared area $\times$ 6 N/mm²

$= 1884.95 \times 6$

$= 11309.7$ N-m

sheared area $= \pi d t$

$= \pi \times \cancel{0.03}^{30} \times 20$

$= 1884.95 \text{ mm}^2$

Total work required/minute = work/hole $\times$ no of holes/min

$= 11309.7 \times 20$

$= 226194$ N-m/min

$= 226194/60$

$\boxed{= 3769.9 \text{ N-m/s}}$

$= \cancel{3769.9/1000}$

$= \cancel{3.7699 \text{ KW}}$

Energy supplies during punching operation which takes 1/10 second

$E_2 = 3769.9 \times \frac{1}{10}$

$\boxed{E_2 = 376.99 \text{ N-m}}$

So maximum fluctuation of energy (ΔE)

$\Delta E = E_1 - E_2 = \frac{1}{2} I(\omega_1^2 - \omega_2^2) = \frac{1}{2} mk^2(\omega_1^2 - \omega_2^2)$

$11309.7 - 376.99 = \frac{1}{2} m \times (1)^2 [(16.75)^2 - (14.66)^2]$

$\boxed{m = 333 \text{ kg}}$

Tus — 1hr final year

" — 3-hr 3rd yr

wed — 5hr final yr.

Radius of Gyration

In terms of mass moment of inertia. It is a perpendicular direction from axis of rotation to a point mass that gives an equivalent inertia to the original object(s)

UNIT-III - Free vibrations.

Longitudinal Vibrations.

Vibration:-

Any motion which repeats itself after an interval of time is called vibration or oscillation or periodic motion.

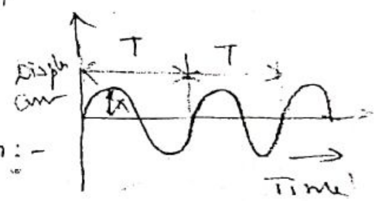
- All bodies possessing mass and elasticity are capable of producing vibrations.
- Vibratory problems, in practice, occur whenever there are rotating or moving parts in a machinery. The study of vibration is concerned with oscillatory motions of the bodies and the forces associated with them.

Causes of vibrations:-

1) Unbalanced forces in the machine and are produced from within the machine itself.
2) Elastic nature of the system.
3) Self excitations produced by the dry friction between the two mating surfaces.
4) External excitations applied on the system
5) Winds may cause the vibrations in certain systems such as transmission and telephone lines under certain conditions
6) Earthquakes also cause vibrations and are greatly responsible for the failure of dams, many buildings etc.

Method of Elimination / Reduction of the undesirable Vibrations :-

* By removing the causes of vibration
* By resting the machinary on proper type of isolators
* By using shock absorbers
* By using dynamic vibration absorbers
* By using the screens (if noise is the objection)



Terms used in vibratory motion:-

(1) Periodic motion:- A motion which repeats itself after equal interval of time.

(2) Time period (t_p):- It is the time taken by a motion to repeat itself. It is also called as period of vibration, and is measured in seconds.

(3) Cycle:- It is the motion completed during one time period.

(4) Frequency (f) :- It is the number of cycles completed in one second. It is expressed in (Hz). It is reciprocal of time period. Mathematically, $f = \frac{1}{t_p}$ Hz.

(5) Natural frequency: Frequency of free vibration of the system.

(6) Amplitude (x): The maximum displacement of a vibrating body from the mean position.

7) Resonance: When the frequency of the external force is equal to the natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This phenomenon is known as resonance.

(8) Damping: It is the resistance to the motion of a vibrating body.

Types of vibratory motion:-

1.) Free (or) Natural vibration:-

If the periodic motion continues after the cause of original disturbance (i.e., initial displacement) is removed, then the body is said to be under free or natural vibration. The frequency of the free vibration is called free or natural frequency.

Ex:- Oscillation of a simple pendulum

2.) Forced vibration:-

When the body vibrates under the influence of external force, then the body is said to be under forced vibrations. The vibrations have same frequency as the applied force.

Ex:- Vibrations in machine tools, electric bells, vibratory conveyors etc..

3) Damped vibration:-

When there is a reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration.

Free damped vibrations: If the damper is connected with free vibrating body to control vibrations, then it is called free damped vibrations.

Forced damped vibrations: If the damper is connected with forced vibrating body to control vibrations, then it is called forced damped vibrations.

Undamped vibrations: The system having no damper is known as undamped vibrations.

Damped free longitudinal vibrations

Damping:-

Damping is defined as the resistance offered by a body to the motion of a vibratory system.

Dampers:-

The device used for the resisting purpose in a vibratory system is called dampers.

Advantage of damping:-

The main adv. of providing damping in mechanical systems is to control the amplitude of vibration so that the failure due to resonance may be avoided.

Types of damping:-

1) Viscous damping
2) Coulomb (or) dry friction damping
3) Solid or structural damping
4) Slip or interfacial damping.

Damping coefficient (C)

The damping force per unit velocity is known as damping co-efficient

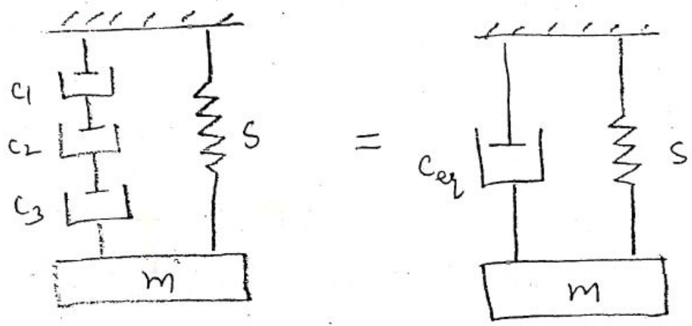
$$C = \frac{\text{damping force}}{\text{velocity}}$$

$$\Rightarrow \boxed{\text{damping force} = C \times \frac{dx}{dt}}$$

Damping in series

The reciprocal of the effective or equivalent damping co-efficient of the dampers in series is the sum of the reciprocal of their individual damping co-efficients.

$$\boxed{\frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}}$$

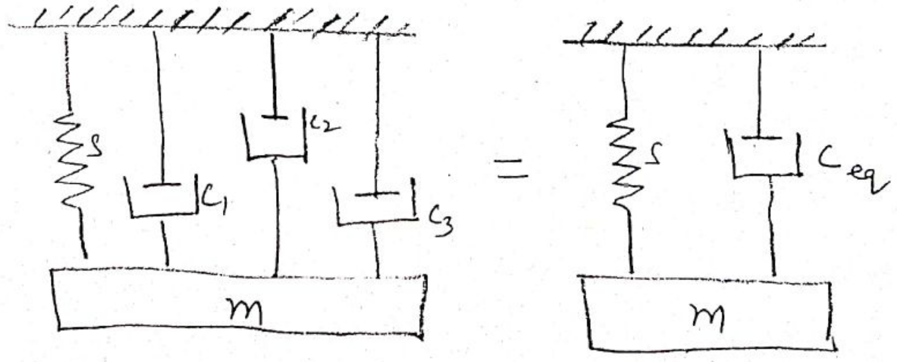


Actual system Equivalent system

Dampers in parallel

The effective damping co-efficient of the dampers in parallel is the sum of their individual damping coefficients.

$$\boxed{c_{eq} = c_1 + c_2 + c_3}$$



Actual system Equivalent system

~~Critical damping coefficient (C_c)~~

Damping factor (or) damping ratio:-

The ratio of actual damping coefficient (c) to the critical damping coefficient (C_c) is called as damping factor or damping ratio.

Mathematically,

$$\text{Damping factor} = \frac{c}{c_c} = \frac{c}{2m\cdot\omega_n}$$

$$C_c = 2m\sqrt{\frac{S}{m}} = 2m\omega_n$$

Logarithmic Decrement:

It is defined as the natural logarithm of the amplitude reduction factor. The amplitude reduction factor is the ratio of any two successive amplitudes on the same side of the mean position in an under damped system.

In general,

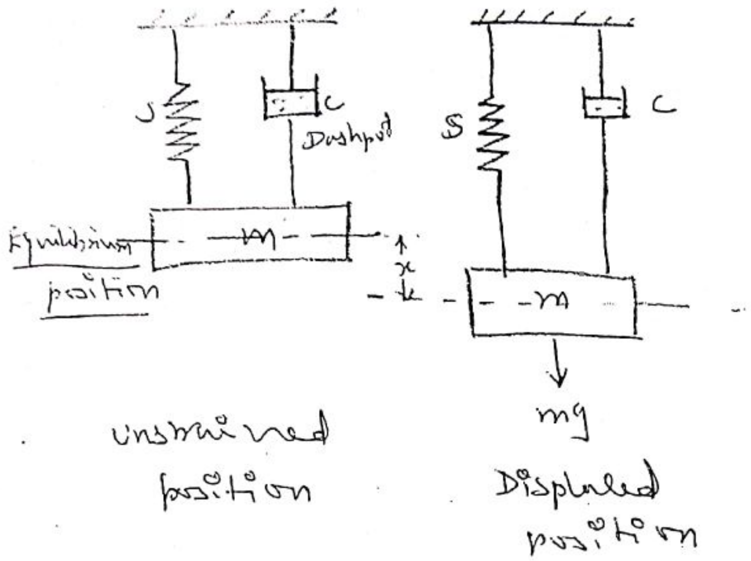
Amplitude reduction factor,

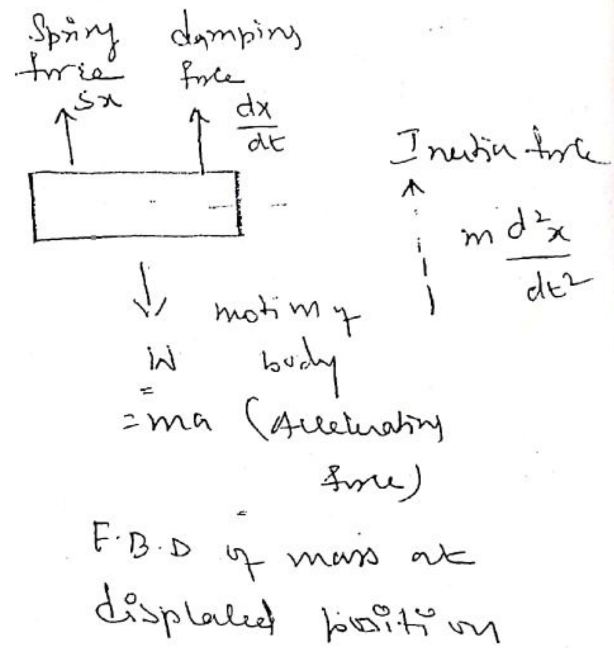
$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \cdots = \frac{x_n}{x_{n+1}} = \text{constant}$$

∴ Logarithmic decrement,

$$\delta = \log_e\left(\frac{x_n}{x_{n+1}}\right)$$

Frequency of Free damped vibrations:-
(Free vibrations with viscous damping)



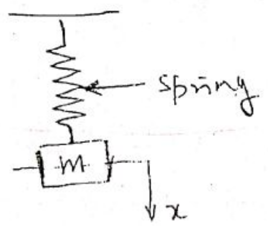


Degree of freedom:

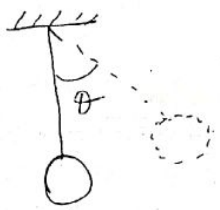
The minimum number of independent co-ordinates required to specify the motion of a system at any instant is known as degrees of freedom of the system. In general, it is equal to the number of independent displacements that are possible. This number varies from zero to infinity.

Discrete systems :-

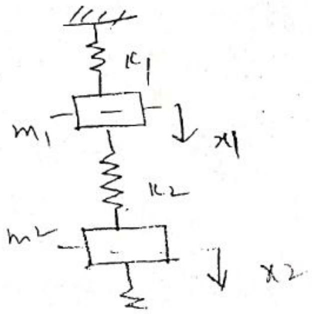
System with finite number of degrees of freedom are called discrete or lumped parameter systems.



(SDOF)
Spring mass



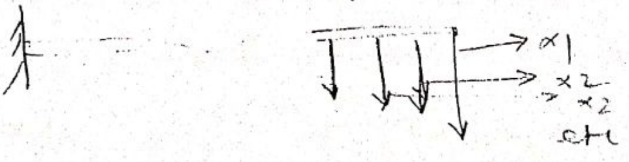
(SDOF)
Simple pendulam



2DOF
Two-mass, two spring system.

Continuous systems:-

System with infinite number of degrees of freedom are called continuous (or) distributed systems. Mostly continuous systems are approximated as discrete system to obtain the solutions easily.



Cantilever Beam (∞DOF).

Types of free Vibrations:-

1. Longitudinal vibrations:-

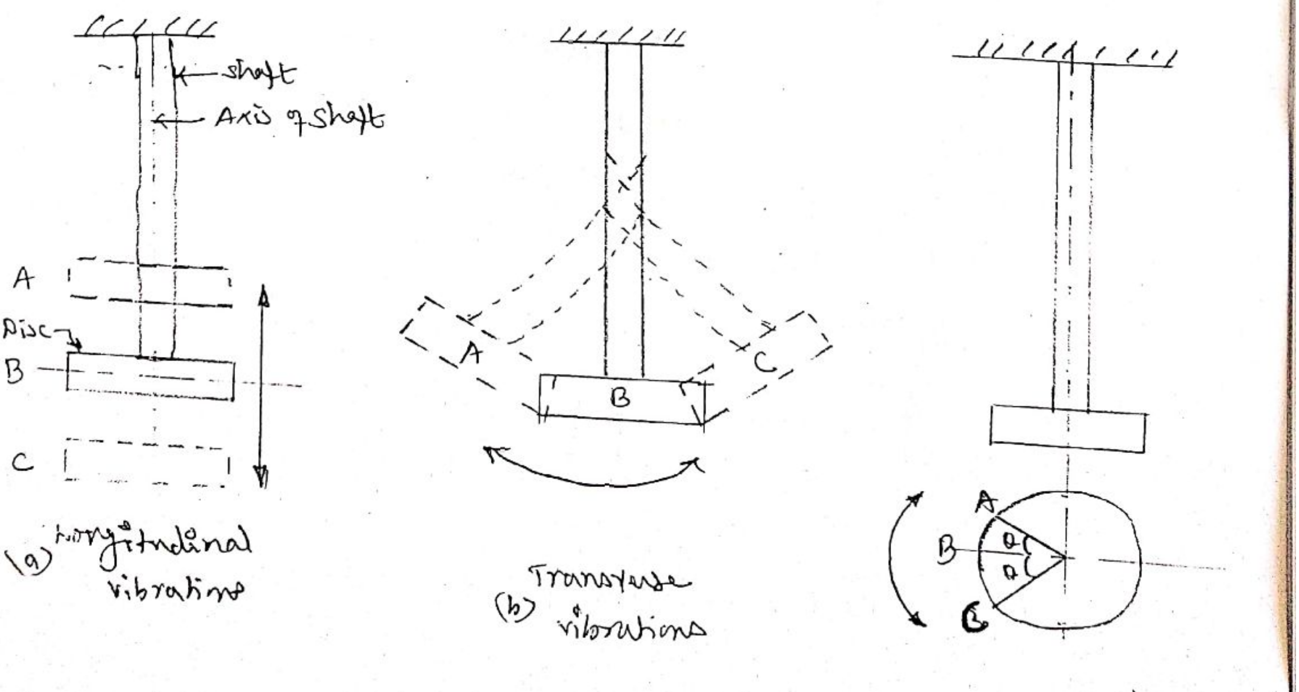
When the particles of the shaft or disc moves parallel to the axis of the shaft, then the vibrations are known as longitudinal vibrations.

2. Transverse Vibrations:-

When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft, then the vibrations are known as transverse vibrations.

3. Torsional vibrations:-

When the particles of the shaft (or) disc move in a circle about the axis of the shaft, then the vibrations are known as torsional vibrations.



(a) Longitudinal vibrations

(b) Transverse vibrations

(c) Torsional vibrations

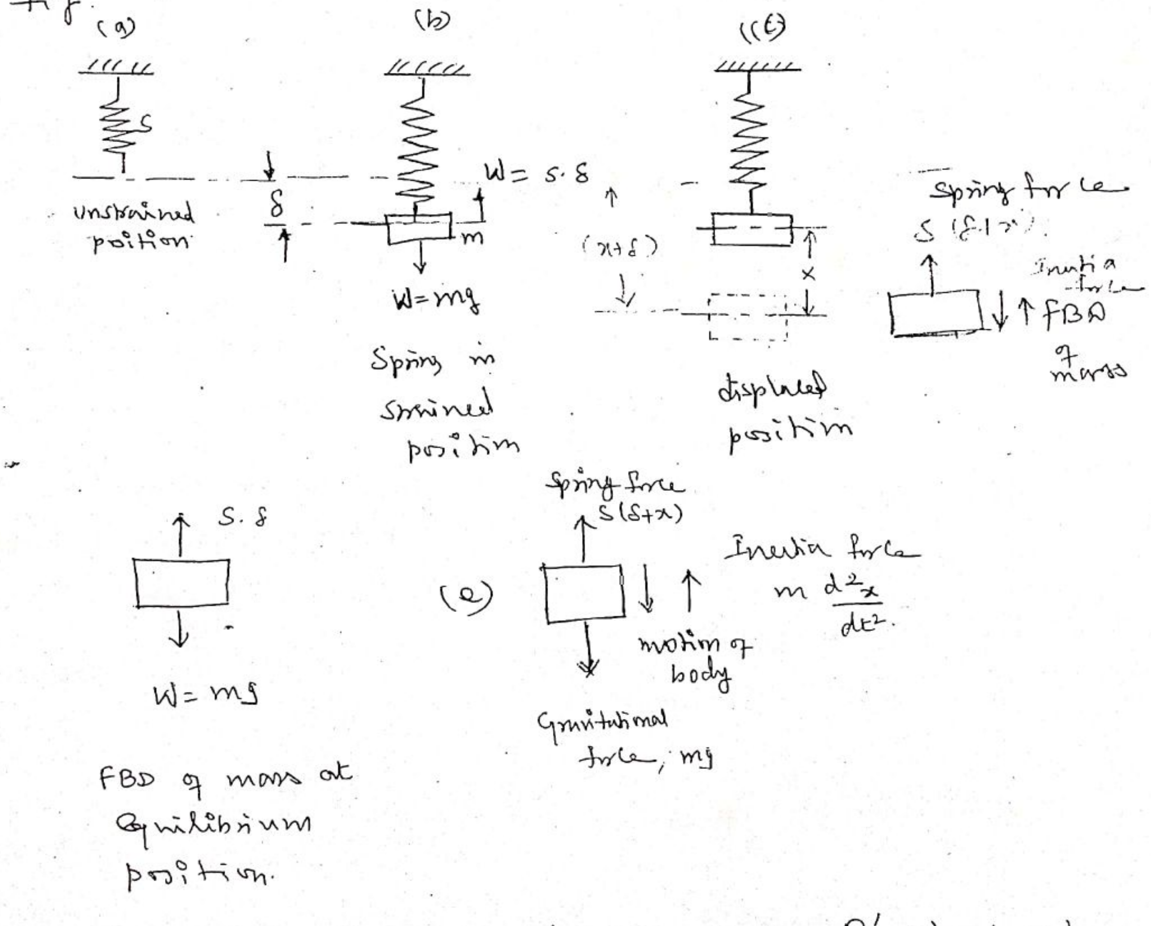
A and C - Extreme positions
B - Mean positions

- **Natural Frequency of Free Longitudinal Vibrations:-**

The natural frequency of the free longitudinal vibrations may be determined by the following three methods.

1) Equilibrium Method (or) Newton's

consider a constraint (i.e spring) of negligible mass in an unstrained position as shown in fig.



This method is based on the D'Alembert's principle that the algebraic sum of the inertia forces and all external forces acting in the vibrating system must be equal to zero to keep the vibrating system in a equilibrium position.

let,

$m \rightarrow$ Mass of the body suspended from the spring

$W = mg$ = Weight of the body suspended from the spring.

$\delta \rightarrow$ Static deflection of the spring due to the external force

$x \rightarrow$ displacement of the body due to external force

$s \rightarrow$ Stiffness of the spring = force required per unit deflection.

In the static equilibrium position,

Upward force = Downward force

$$\boxed{s.\delta = mg}$$

Now, the mass 'm' is displaced down through a distance x, as shown in fig (c) Then the forces acting on the mass is shown in fig (e). are,

Accelerating force $= m \times a = m\frac{d^2x}{dt^2}$ (downwards)

Inertia force = − Accelerating force

$= m.\frac{d^2x}{dt^2}$ (upwards)

Spring force $= W - s(\delta + x)$

$= W - s\delta - sx$

$= \cancel{s\delta} - \cancel{s\delta} - sx$

$= -sx$

* According to D' Alembert's principle

Inertia force + Σ external forces = 0.

$$m \cdot \frac{d^2x}{dt^2} + S \cdot x = 0.$$

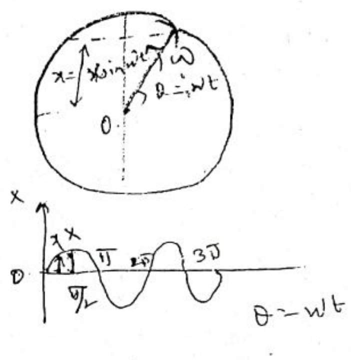
$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + \frac{S}{m} \cdot x = 0} \rightarrow (1).$$

This is the differential equation of motion for a single degree of freedom spring-mass system having free vibrations. This equation is also known as governing equation (or) equation of motion of free undamped longitudinal vibration.

Natural frequency of the spring-mass system

We know that the fundamental equation of simple harmonic motion is,

$$\boxed{\frac{d^2x}{dt^2} + \omega_n^2 x = 0.} \rightarrow (2)$$



$$\boxed{\omega_n = \sqrt{\frac{S}{m}}}$$

Where,

$\omega_n \rightarrow$ circular frequency of the vibrating system.

Natural frequency,

Time period, $t_p = \frac{2\pi}{\omega}$

$$\boxed{f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{S}{m}}}$$

(or)

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}} \qquad \boxed{\because\ S\cdot\delta = m\cdot g}$$

(or)

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\delta}\ HZ.}$$

The value of 'δ' may be found out from the given conditions of the problem. For longitudinal vibrations, it may be obtained by the relation,

$$\frac{Stress}{strain} = E \Rightarrow \frac{W}{A} \times \frac{l}{\delta} = E$$

$$\Rightarrow \boxed{\delta = \frac{W \times l}{AE}}$$

Problem:-

1) Stiffness of a closely coiled helical Spring is 9.2 N/mm find its natural frequency of longitudinal vibrations of a tensile mass of 17.5 kg is hung from it.

Given:-

$S = 9.2\ N/mm = 9.2 \times 10^3\ N/m$

$m = 17.5\ kg,$

To find:- $f_n = ?$

Soln:-

W.K.T $f_n = \frac{1}{2\pi}\sqrt{\frac{S}{m}} = \frac{1}{2\pi}\sqrt{\frac{9.2\times10^3}{17.5}}$

$$\Rightarrow \boxed{f_n = 3.649\ HZ}$$

1) A shaft of 100 mm diameter and 1 metre long is fixed at one end and other end carries a flywheel of mass 1 tonne. Taking young's modulus for the shaft material as 200 GN/m^2, find the frequency of longitudinal vibrations.

Given:-

$d = 100\ mm = 0.1\ m$; $l = 1\ m$; $m = 1$ tonne $= 1000\ kg$

$E = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$.

To find:- Natural frequency 'f_n' = ?

Soln:-

Static deflection of the shaft,

$$\delta = \frac{W \times l}{A \cdot E} = \frac{(1000 \times 9.81) \times 1}{\frac{\pi}{4}(0.1)^2 \times 200 \times 10^9} = \underline{\underline{6.245 \times 10^{-6}\ m}}$$

The natural frequency of longitudinal vibrations is given by,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{6.245 \times 10^{-6}}} = \underline{\underline{199.47\ HZ}}.$$

2) A steel wire ($E = 1.96 \times 10^{11}\ N/m^2$) is of 2 mm diameter and is 30 metres long. It is fixed at the upper end and carries a mass 'm' kg at its lower end. Find 'm' so that the frequency of longitudinal vibrations is 4 cycles/sec.

Given:-

$E = 1.96 \times 10^{11}\ N/m^2$; $d = 2\ mm = 2 \times 10^{-3}\ m$;

$l = 30\ m$; $f_n = 4$ cycles/sec $= 4\ Hz$.

To find:- 'm' = ?

Soln:-

w.k.t, $f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

$$4 = \frac{1}{2\pi}\sqrt{g/\delta}$$

Static deflection, $\boxed{\delta = 0.01553\ m}$

Also,

$$\delta = \frac{W \times l}{A \cdot E} = \frac{m \cdot g \cdot l}{A\ E}$$

$\Rightarrow A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (2 \times 10^{-3})^2 = 3.1416 \times 10^{-6}\ m^2$

$$0.01553 = \frac{m \times 9.81 \times 30}{3.1416 \times 10^{-6} \times 1.96 \times 10^{11}}$$

$\Rightarrow$ $\boxed{\text{Mass } m = 32.49\ kg.}$

4) A Cantilever shaft 50 mm diameter and 300 mm long has a disc of mass 100 kg at its free end. The Young's modulus for the shaft material is $200\ GN/m^2$. Determine the frequency of longitudinal and transverse vibrations of the shaft.

Soln:-

$d = 50\ mm = 0.05\ m$; $l = 300\ mm = 0.03\ m$;

$m = 100\ kg$; $E = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$

To find:-

Frequency of longitudinal & Transverse vibrations.

Soln:-

C.S Area of shaft, $A = \frac{\pi d^2}{4} = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3}\ m^2$

moment of Inertia, of the shaft,

$$I = \frac{\pi d^4}{64} = \frac{\pi}{64}(0.05)^4 = 0.3 \times 10^{-6}\ m^4.$$

Frequency of longitudinal vibration $\qquad \frac{Wl^3}{3EI}$

Static deflection of the shaft,

$$\delta = \frac{W \times l}{A \cdot E} = \frac{100 \times 9.81 \times 0.3}{1.96 \times 10^{-3} \times 200 \times 10^9} = 0.751 \times 10^{-6}\ m$$

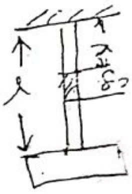
$(\because W = mg)$

$\therefore$ frequency of longitudinal vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.751 \times 10^{-6}}} = 575\ Hz.$$

Effect of Inertia of the Constraint in Longitudinal vibrations:-

In deriving the expressions for natural frequency of longitudinal vibrations & Transverse vibrations, we have neglected the inertia of the Constrain (i.e; Shaft (or Spring). When the inertia of the constraint is taken into account.



Then,

Natural frequency, $\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{S}{m+\frac{m_c}{3}}}}$

$m \rightarrow$ mass of the body

$m_c \rightarrow$ mass of the Constraint

Stiffness of a closed coiled helical spring is 9.2 N/m. Find its natural frequency of longitudinal vibrations if a tensile mass of 17.5 kg is hung from it and if the mass of the spring is 6 kg.

Given:- $m = 17.5$ kg, $m_c = 6$ kg, $S = 9.2$ N/mm

To find: f_n - ?

Soln:-

$$f_n = \frac{1}{2\pi}\sqrt{\frac{S}{m+\frac{m_c}{3}}}$$

$$\Rightarrow \boxed{f_n = 3.46 \text{ Hz}}$$

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1). A vibrating system consists of a mass of 200 kg, a spring of stiffness 80 N/mm and a damper with damping coefficient of 800 N/m/s. Determine the frequency of vibration of the system.

Given:-

$m = 200$ kg ; $s = 80$ N/mm $= 80 \times 10^3$ N/m ; $C = 800$ N/m/s

To find:- frequency of damped vibration

Soln:-

frequency of damped vibration } $f_d = \frac{\omega_d}{2\pi}$ $\zeta \rightarrow$ zeta

$$\omega_d = \sqrt{1-\zeta^2} \cdot \omega_n$$

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{80 \times 10^3}{200}} = 20 \text{ rad/s}$$

$$\zeta = \frac{c}{c_c} = \frac{\cancel{800}^{4}}{2 \times \cancel{200} \times 20} = 0.1$$

$$\therefore \boxed{\omega_d = 19.899 \text{ rad/s}}$$

$$\therefore f_d = {}^{19.899}/_{2\pi} = 3.17 \text{ HZ.}$$

2.) An instrument vibrates with a frequency of 1.24 HZ when there is no damping. When the damping is provided, the frequency of damped vibration was observed to be 1.03 HZ. Find: (i) the damping factor and (ii) the logarithmic decrement.

Given:-

$f_n = 1.24$ HZ ; $f_d = 1.03$ HZ

To find:-

(i) damping factor 'ζ' (ii) logarithmic decrement 'δ'

Soln:- Natural circular frequency of undamped vibrations,

$$\omega_n = f_n \times 2\pi = 1.24 \times 2\pi = 7.791 \text{ rad/s}$$

circular frequency of damped vibrations,

$W_d = f_d \times 2\pi = 1.03 \times 2\pi = 6.472$ rad/s

W.K.T,

$$W_d = \sqrt{1-\xi^2} \times \omega_n$$

$\Rightarrow$ $\boxed{\text{damping factor } \xi = 0.556}$

And also,

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi \times 0.556}{\sqrt{1-0.556^2}}$$

$\Rightarrow$ $\boxed{\text{logarithmic decrement } \delta = 4.21}$

3.) A mass suspended from a helical spring vibrate in a viscous fluid medium whose resistance varies directly with the speed. It is observed that the frequency of damped vibration is 90 per minute and that the amplitude decreases to 20% of its initial value in one complete vibrations. Find the frequency of the free undamped vibration of the system.

Given:- $f_d = 90/min = 1.5/sec$; $x_1 = 0.20 x_0$

To find:- $f_n = ?$

Soln:-

W.K.T, $f_n = \frac{\omega_n}{2\pi}$; $\frac{x_0}{x_1} = 1/0.20 = 5$

logarithmic decrement,

$$\delta = \ln\left(\frac{x_0}{x_1}\right) = \ln(5) = 1.609$$

Also,

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$1.609 = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \Rightarrow \xi = 0.247$$

W.K.T

$$\omega_d = \sqrt{1-\xi^2}\ \omega_n \qquad \omega_d = 2\pi \times f_d$$
$$= 9.42 \text{ rad/sec}$$

$$\Rightarrow \boxed{\omega_n = 9.72 \text{ rad/s}}$$

$$\therefore f_n = \frac{\omega_n}{2\pi} = \frac{9.72}{2\pi} = 1.548 \text{ HZ}.$$

4) A vibrating system consists of a mass of 8 kg, spring of stiffness 5.6 N/mm and a dashpot of damping coefficient of 40 N/m/s. Find:
(a) the critical damping coefficient
(b) the damping factor (c) the natural frequency of damped vibration (d) the logarithmic decrement,
(e) the ratio of two consecutive amplitudes and
(f) the number of cycles after which the original amplitude is reduced to 20 percent.

Given:- $m = 8$ kg ; $s = 5.6$ N/mm $= 5.6\times10^3$ N/m; $c = 40$ N/m/s

To find:- (a) C_c (b) ξ (c) f_d (d) δ (e) $\frac{x_n}{x_{n+1}}$ (f) n

Soln:-

a) Critical damping coefficient

$$C_c = 2m\omega_n = 2m\sqrt{\frac{s}{m}} = 423.32 \text{ N/m/s}$$

b) Damping factor

$$\xi = \frac{c}{c_c} = 0.0945$$

c) Natural frequency of damped vibrations,

$$f_d = \frac{\omega_d}{2\pi} \qquad \omega_d = \sqrt{1-\xi^2}\ .\ \omega_n$$
$$= 26.34 \text{ rad/s}$$

$$\Rightarrow f_d = 4.19 \text{ HZ}$$

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d) Logarithmic decrement

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \underline{0.596}$$

e) Ratio of two consecutive amplitudes

W.K.T, $\delta = \ln\left[\frac{x_n}{x_{n+1}}\right] \Rightarrow \frac{x_n}{x_{n+1}} = e^{\delta} = e^{0.596}$

$$\rightarrow \frac{x_n}{x_{n+1}} = \underline{1.815}$$

f) No. of cycles after which the amplitude is reduced to 20%.

x_0 = Amplitude at starting position,

x_n = Amplitude after 'n' cycle $= 20\% \, x_0$

$= 0.2 \, x_0$

W.K.T,

$$\delta = \frac{1}{n} \ln\left(\frac{x_0}{x_n}\right)$$

$$0.596 = \frac{1}{n} \ln(5)$$

$$\Rightarrow n = \underline{2.7 \text{ cycles}}$$

5) A body of mass of 50 kg is supported by an elastic structure of stiffness 10 kN/m. The motion of the body is controlled by a dashpot such that the amplitude of vibration decreases to one-tenth of its original value after two complete vibrations. Determine:

(i) the damping ratio, (ii) the damping force at 1 m/s and (iii) the natural frequency of vibration.

Given:- $m = 50$ kg ; $S = 10$ kN/m $= 10 \times 10^3$ N/m ,
$n = 2$. $x_2 = \frac{1}{10} \times x_0$, $v = 1$ m/s

To find:- (i) ζ (ii) damping force (iii) f_d.

Soln:-

(i) damping factor:

x_0 = Initial Amplitude

x_2 = Final amplitude after two complete vibrations. $= 0.1\, x_0$.

for 'n' cycles logarithmic decrement,

$$\delta = \frac{1}{n} \ln\left[\frac{x_0}{x_n}\right] = \frac{1}{2} \ln\left(\frac{x_0}{x_2}\right)$$

$$\Rightarrow \delta = \frac{1}{2} \ln\left(\frac{x_0}{0.1\, x_0}\right) = \underline{1.151}$$

We also know that,

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 1.151$$

$$\Rightarrow \boxed{\zeta = 0.18}$$

(ii) damping force at 1 m/s:

W.K.T

$$\zeta = \frac{c}{c_c}$$

$$\Rightarrow c = \zeta \times c_c$$

$$= \underline{254.56 \text{ N/m/s}}$$

$$c_c = 2m\omega_n$$
$$= 2m\sqrt{\frac{S}{m}}$$
$$= 2\sqrt{S \times m}$$
$$= \underline{1414.21 \text{ N/m/s}}$$

$\therefore$ damping coefficient, c = damping force / velocity

$$\Rightarrow \boxed{\text{damping force} = 254.56 \text{ N}}$$

(iii) Natural frequency of vibration

W.K.T,

$$\omega_d = \sqrt{1-\zeta^2} \times \omega_n \qquad \Big| \; \omega_n = \sqrt{\frac{S}{m}}$$

$$= 13.91 \text{ rad/s}$$

$$\therefore f_n = \frac{\omega_d}{2\pi} = \frac{13.91}{2\pi} = 2.214 \text{ Hz.}$$

b) In a single degree damped vibrating system, the suspended mass of 3.75 kg makes 12 osillations in 7 seconds when disturbed from its equilibrium position. The amplitude decreases to 0.33 of the initial value after 4 osillations. Determine : (i) the stiffness of the spring, (ii) the logarithmic decrement, (iii) the damping factor, and (iv) damping co-efficient.

Given:- $m = 3.75$ kg ; $N = 12$; $t = 7$ s ; $x_4 = 0.33\, x_1$

To find:- (i) S (ii) δ (iii) ζ (iv) C.

Soln:-

Since 12 osillations are made in 7 seconds, therefore the frequency of free vibration,

$$f_n = \frac{12}{7} = 1.7143 \text{ Hz}$$

$\therefore$ circular natural frequency,

$$\omega_n = f_n \times 2\pi = 10.77 \text{ rad/s}$$

(i) Stiffness of spring.

W.K.T $\omega_n = \sqrt{S/m} \Rightarrow S = 435.01$ N/m.

let,

x_0 = Initial amplitude

x_4 = Final amplitude after four oscillations

= $0.33\, x_0$

for 'n' cycles we know that the logarithmic decrement,

$$\delta = \frac{1}{n} \ln\left[\frac{x_0}{x_n}\right] = \frac{1}{4} \ln\left[\frac{x_0}{0.33x_0}\right]$$

$$\Rightarrow \boxed{\delta = 0.277}$$

(iii) <u>damping factor</u>

W.K.T,

$$\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}} = 0.277$$

$$\Rightarrow \boxed{\zeta = 0.044}$$

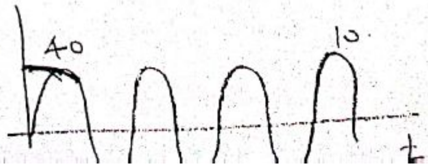
(iv) <u>Damping coefficient</u>

W.K.T

$$\zeta = \frac{c}{c_c} \qquad \Big|\; c_c = 2m\omega_n$$

$$\Rightarrow \boxed{C = 3.566 \text{ N/m/s}}$$

7) A machine mounted on springs and fitted with a dashpot has a mass of 100 kg. There are four springs, each of stiffness 25 kN/m. The amplitude of vibrations reduces from 40 mm to 10 mm in three complete oscillations. Assuming that the damping force varies as the velocity, determine:



(i) the resistance of dashpot at unit velocity
(ii) the ratio of frequencies of damped and undamped vibrations, and
(iii) the periodic time of damped vibrations.

Given:-

$m = 100$ kg ; $s = 25$ kN/m $= 25\times10^3$ N/m ;
No. of springs $= 4$; $x_0 = 40$ mm ; $x_3 = 10$ mm.

To find:- (i) c (ii) f_d/f_n (iii) t_d.

Soln:-

Since the stiffness of each spring is 25×10^3 N/m and there are four springs,

$\therefore$ Total stiffness $= 4\times 25\times10^3 = 100\times10^3$ N/m.

(i) Resistance of the dashpot at unit velocity (c)

let,
x_0 = Initial amplitude
x_3 = final amplitude after three complete vibrations

For n cycles, we know that the logarithmic decrement,

$$\delta = \frac{1}{n}\ln\left(\frac{x_0}{x_n}\right) = \frac{1}{3}\ln\left(\frac{x_0}{x_3}\right) = \frac{1}{3}\ln\left(\frac{40}{10}\right)$$

$$\Rightarrow \quad \delta = \underline{\underline{0.462}}$$

Also,

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 0.462$$

On solving we get, $\zeta = \underline{\underline{0.0733}}$

$$\zeta = \frac{c}{c_c} = \frac{c}{2m\,\omega_n} \qquad \Bigg|\ \omega_n = \sqrt{\frac{s}{m}}$$

$$\Rightarrow \quad \boxed{c = 463.81 \text{ N/m/s.}}$$

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(ii) Ratio of the frequency of the damped vibration to the frequency of undamped vibration:

W.K.T,
$$\frac{f_d}{f_n} = \frac{\omega_d}{2\pi} \times \frac{2\pi}{\omega_n} = \frac{\omega_d}{\omega_n}$$

$$\Rightarrow \frac{f_d}{f_n} = \frac{\sqrt{1-\xi^2} \times \omega_n}{\omega_n} = \sqrt{1-\xi^2}$$

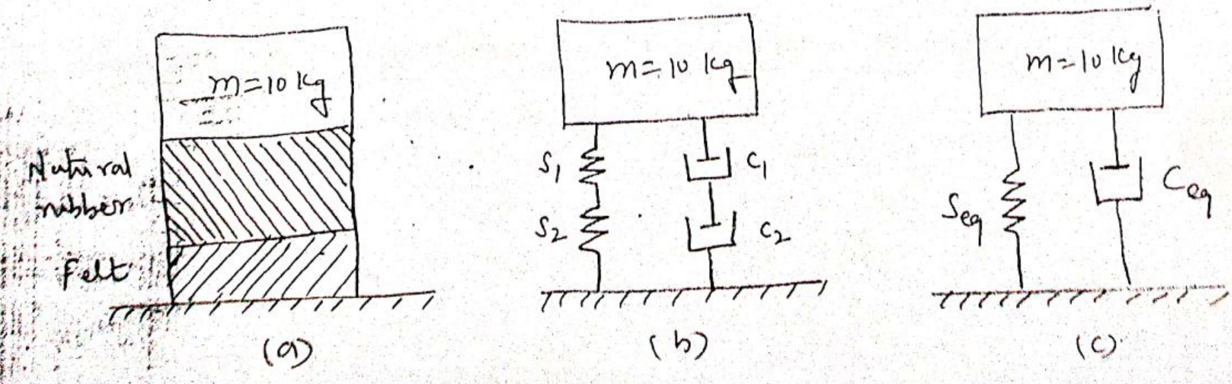
$$\Rightarrow \boxed{\frac{f_d}{f_n} = 0.9973}$$

(iii) Periodic time of damped vibration (t_d)

$$t_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{1-\xi^2} \cdot \omega_n}$$

$$\Rightarrow \boxed{t_d = 0.199\ s.}$$

8) Between a solid mass of 10 kg and the floor are kept two slabs of isolators, natural rubber and felt, in series. The natural rubber slab has a stiffness of 3000 N/m and an equivalent viscous damping coefficient of 100 N-sec/m. The felt has a stiffness of 12000 N/m and equivalent viscous damping coefficient of 330 N-sec/m. Determine the undamped and the damped natural frequencies of the system in vertical direction, neglecting the mass of the isolators.



Given:- $m = 10$ kg ; $S_1 = 3000$ N/m ; $C_1 = 100$ N-sec/m

$S_2 = 12000$ N/m ; $C_2 = 330$ N-sec/m

To find:- (i) f_n (ii) f_d.

Soln:-

The fig (a) depicts the given system. The isolators being in series, the system can be systematically represented by fig (b) & fig (c). represents the reduced system having a equivalent spring and damper.

Equivalent spring stiffness (S_{eq}) when the springs are in series,

$$\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{3000} + \frac{1}{12000}$$

$$\Rightarrow S_{eq} = 2400 \text{ N/m}$$

Equivalent damping co-efficient (C_{eq}) when the dampers are in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{100} + \frac{1}{330}$$

$$\Rightarrow C_{eq} = 76.744 \text{ N-sec/m}$$

(a) Undamped natural frequency of the system (f_n)

W.K.T $f_n = \frac{\omega_n}{2\pi} = 2.465$ HZ | $\omega_n = \sqrt{\frac{S_{eq}}{m}}$

(b) Damped natural frequency of the system (f_d)

W.K.T $f_d = \frac{\omega_d}{2\pi}$ | $\omega_d = \sqrt{1-\xi^2} \times \omega_n$

$\Rightarrow \omega_d = 15$ rad/s | $\xi = \frac{C_{eq}}{C_c} = \frac{C_{eq}}{2m\omega_n}$

$\therefore f_d = 2.388$ HZ

(9) A machine weighs 18 kg and is supported on springs and dashpots. The total stiffness of the springs is 12000 N/m and damping ratio is 0.2 N/mm/s. The system is initially at rest and a velocity of 120 mm/s is imparted to the mass. Determine:

(i) the displacement and velocity of mass as a function of time, and

(ii) the displacement and velocity after 0.4 s.

<u>Given:-</u> $m = 18\,kg$; $S = 1200\,N/m$; $C = 0.2\,N/mm/s$

(or) $C = 200\,N/m/s$; $v = 120\,mm/s = 0.12\,m/s$

<u>To find:-</u> (i) x & v as a function of time 't'.

(ii) x & v after 0.4 s.

<u>Soln:-</u>

Let us find the value of ξ, so that to identify the type of damping.

$$\xi = \frac{C}{C_c} = \frac{C}{2m\omega_n} = \frac{C}{2m\sqrt{S/m}} = \frac{C}{2\sqrt{S \times m}}$$

$$\Rightarrow \xi = \underline{0.215} < 1$$

Since, $\xi < 1$, the system is under damped.

(i) <u>displacement and velocity of mass as a function of time:</u>

W.K.T, the equation for displacement for a under damped system,

$$\boxed{x = X \cdot e^{-\xi \omega_n t} \cdot \sin(\omega_d \cdot t + \phi)}$$

Here, X & ϕ are arbitrary constants.

Here,

$$\omega_d = \sqrt{1-\xi^2} \times \omega_n$$

$$= \sqrt{1-(0.215)^2} \times 25.82$$

$$\Rightarrow \omega_d = \underline{\underline{25.2 \text{ rad/s}}}$$

Substituting the values of ξ and ω_n, we get

$$x = X \cdot e^{-0.215 \times 25.82 t} \cdot \times \sin(25.2t + \phi)$$

$$x = X \cdot e^{-5.55t} \sin(25.2t + \phi) \longrightarrow (2)$$

Apply initial conditions to the above equation to find X and ϕ values,

(i) At $t = 0$, $x = 0$ So eqn (2) becomes,

$$0 = X \cdot (1) \sin(0 + \phi)$$

$$\Rightarrow X \sin\phi = 0 \quad (\text{or}) \quad \sin\phi = 0. \quad \because X \neq 0.$$

$$\Rightarrow \boxed{\phi = 0}$$

(ii) At $t = 0$, $V = \frac{dx}{dt} = 0.12 \text{ m/s}.$

$$V = \frac{d}{dt}\left(X e^{-5.55t} \sin 25.2t\right)$$

$$= \left\{(X \cdot e^{-5.55t}) \left[25.2 \times \cos 25.2t\right]\right\} +$$

$$\left\{(X \sin 25.2t)(-5.55 t \times e^{-5.55t})\right\} \longrightarrow (3)$$

Applying the second initial condition we get,

$$0.12 = 25.2 X + 0$$

$$\Rightarrow \boxed{X = 4.762 \times 10^{-3} \text{ m}}$$

Substituting the values of X & ϕ in eqn 2 & 3 we get,

(2) ⇒ displacement,

$$x = (4.762 \times 10^{-3}\, e^{-5.55t} \sin 25.2\, t)\ m \longrightarrow (4)$$

(3) ⇒ Velocity,

$$V = 0.12\, e^{-5.55t} \cdot \cos 25.2t - 0.026 \cdot e^{-5.55t} \cdot \sin 25.2t$$

$$V = e^{-5.55t} [0.12 \cos 25.2t - 0.026 \sin 25.2t]\ m/s. \longrightarrow (5)$$

(ii) Displacement and Velocity after 0.4 s.

Substituting $t = 0.4\,s$ in eqns (4) & (5) we get,

(4) ⇒

$$x = 4.762 \times 10^{-3}\, e^{-5.55 \times 0.4} \sin \left(25.2 \times 0.4 \times \frac{180}{\pi}\right)$$

$$\Rightarrow \quad x = 3.151 \times 10^{-4}\ m$$

(5) ⇒

$$v = e^{-5.55 \times 0.4} \left[0.12 \cos \left(25.2 \times 0.4 \times \frac{180}{\pi}\right) - 0.026 \sin \left(25.2 \times 0.4 \times \frac{180}{\pi}\right)\right]$$

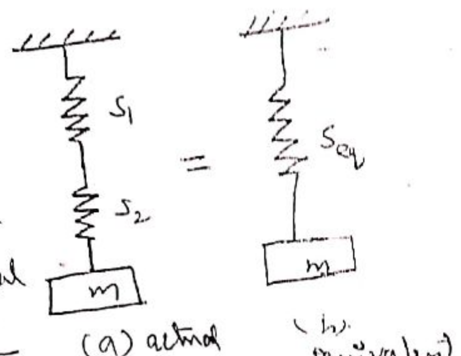
$$\Rightarrow \quad v = -8.614 \times 10^{-3}\ m/s.$$

Determination of Equivalent spring stiffness

So far, we have discussed the system having only one spring. But in practice, springs in series and springs in parallel are mostly used. In order to find the natural frequency of the above systems, first we have to determine the equivalent spring stiffness.

Springs in series:-

Refer fig (a). Each spring is subjected to same load applied at the end of one spring. It can be noted that the total deflection of the assembly is equal to the algebraic sum of the deflections of the two springs.



(a) actual system (b) equivalent system

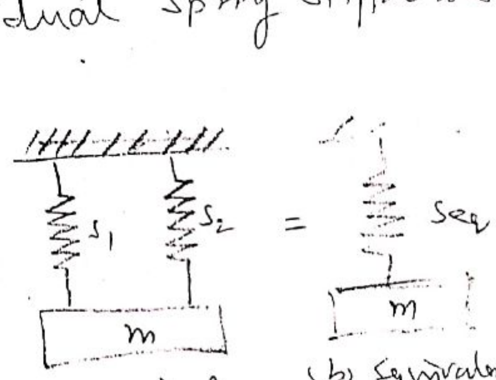
$$\delta = \delta_1 + \delta_2$$

(or) $$\frac{W}{S_{eq}} = \frac{W}{S_1} + \frac{W}{S_2} \Rightarrow \boxed{\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2}}$$

Thus, in case of series springs, the reciprocal of the equivalent spring stiffness is equal to the sum of the reciprocals of individual spring stiffnesses.

Springs in parallel:-

In this case, the extension of each spring is the same. The total load will be shared by individual springs.



(a) actual system (b) Equivalent systems

$$W = W_1 + W_2$$

$$\delta \cdot S_{eq} = \delta \cdot S_1 + \delta \cdot S_2$$

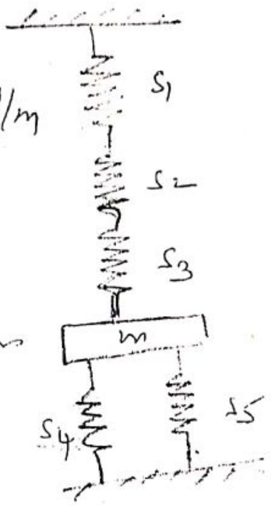
$$\boxed{S_{eq} = S_1 + S_2}$$

Thus in this case, equivalent spring stiffness is the sum of the individual spring stiffness

Problem:-

1) For the system shown in fig $S_1 = S_2 = 500$ N/m $S_3 = 1500$ N/m, $S_4 = 3000$ N/m ; $S_5 = 2000$ N/m. Find the mass 'm' such that the system has a natural frequency of 6.75 Hz.

Given:



$S_1 = S_2 = 500$ N/m ; $S_3 = 1500$ N/m ; $S_4 = 3000$ N/m

$S_5 = 2000$ N/m ; $f_n = 6.75$ Hz.

Soln:-

If S_{e1} is the effective spring stiffness of the top three springs in series, then

$$\frac{1}{S_{e1}} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} = \frac{1}{500} + \frac{1}{500} + \frac{1}{1500}$$

$$\Rightarrow \boxed{S_{e1} = 214.286 \text{ N/m}}$$

If S_{e2} is the effective spring stiffness of the lower two springs in parallel, then

$$S_{e2} = S_4 + S_5$$

$$= 3000 + 2000$$

$$\Rightarrow \boxed{S_{e2} = 5000 \text{ N/m}}$$

The natural frequency,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{S_{eq}}{m}}$$

$$6.75 = \frac{1}{2\pi}\sqrt{\frac{5214.284}{m}}$$

Mass, $\boxed{m = 2.899 \text{ kg}}$

Transverse Vibrations

Definition:

When the particles of the shaft or disc move approximately $\perp^r$ to the axis of the constraint (i.e. shaft), then the vibrations are termed as torsional vibrations. In this case, the shaft bends and straight ~~away~~ alternately.

Example: vibrations of beams (Cantilever, simply supported, fixed)

Natural frequency of free Transverse vibrations

Natural frequency, $f_n = \frac{1}{2\pi}\sqrt{g/\delta}$

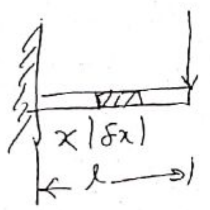
(or)

$$f_n = \frac{0.4985}{\sqrt{\delta}}, \text{ Hz}$$

$\delta \rightarrow$ static deflection of beam in 'm'

Natural frequency of free transverse vibrations by considering the effect of inertia of the constraint

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{33}{140} m_c}} \text{ Hz}$$



for a beam with one end is fixed and other end is free

k - stiffness in 'm'

m_c - mass of constraint 'kg'.

$m_c = m_s \times l$.

for fixed beam with with Centre point load,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{13}{35} m_c}}$$

For simply supported beam with point load.

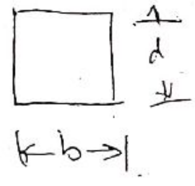
$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{17}{35} m_c}}$$

1) A steel bar 25 mm wide, 45 mm deep and 900 mm long is fixed at one end and the other carries a load of mass 600 kg. Find the natural frequency of transverse vibrations. Take the young's modulus for the shaft material as $210\ GN/m^2$. If an additional mass of 200 kg is distributed uniformly over the length of the shaft, what will be the frequency of vibrations?

Given: $b = 25mm = 25\times10^{-3}\ m$; $d = 45\ mm = 45\times10^{-3}\ m$

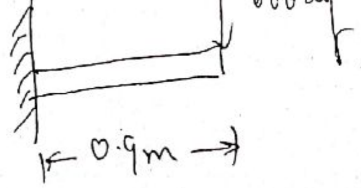
$l = 900\ mm = 0.9m$; $m = 600\ kg$;

$E = 210\ GN/m^2 = 210\times10^9\ N/m^2$.



(i) Natural frequency of transverse vibration (f_n)

$$f_n = \frac{0.4985}{\sqrt{\delta}}$$



for beam one end fixed and another end is carrying point load,

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deflection

$$\delta = \frac{WL^3}{3EI} \qquad W = m \times g$$

where

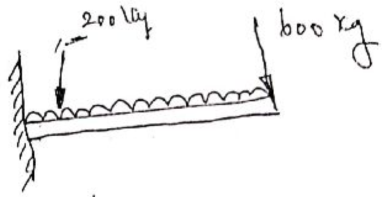
'I' (for rectangular beam) $= \dfrac{bd^3}{12}$

$$\Rightarrow \boxed{I = 1.898 \times 10^{-7}\ m^4}$$

$$\therefore \delta = \frac{(600 \times 9.81) \times 0.9^3}{3 \times 210 \times 10^9 \times 1.898 \times 10^{-7}} = \underline{\underline{0.03587\ m}}$$

$$\therefore f_n = \frac{0.4985}{\sqrt{0.03587}} = \underline{\underline{2.632\ Hz}}$$

(ii) <u>Natural frequency of transverse vibration taking mass of the shaft into account</u>



We know that natural frequency of transverse vibration taking mass of the shaft into account,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{33}{140} m_c}} \qquad \begin{array}{|l} m = 600\ kg \\ m_c = 200\ kg \end{array}$$

$$\text{Stiffness, } k = \frac{W}{\delta} = \frac{(600 \times 9.81)}{0.035872} = \underline{164083.4\ N/m}$$

$$f_n = \underline{\underline{2.534\ Hz}}$$

Natural Frequency of Free Transverse Vibrations for a shaft with number of point loads - Multi degree freedom

There are two methods by which natural frequency of transverse vibration can be found out when the shaft carries various point loads.

(i) **Energy Method**

This method gives accurate results but involves tough calculations, if there are many loads.

(ii) **Dunkerley's method**

This method is semi-empirical. This gives approximate results but is simple.

1) A shaft 180 mm diameter is supported in two bearings 2.5 m apart. It carries three discs of mass 250 kg, 500 kg and 200 kg at 0.6 m, 1.5 m and 2 m from the left end. Determine the natural frequency of transverse vibrations. Take modulus of elasticity for the shaft material as 211 GN/m².

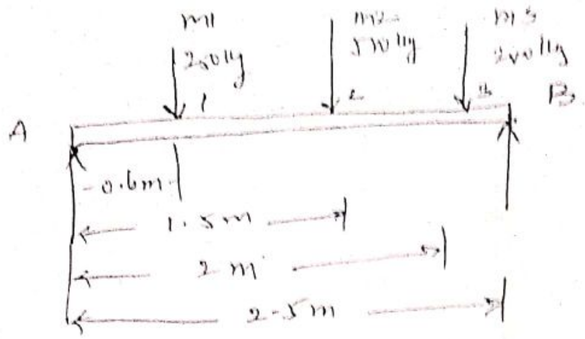
Given:- $d = 180\,mm = 0.18\,m$; $l = 2.5\,m$;

$m_1 = 250\,kg$; $m_2 = 500\,kg$; $b_2 = 1\,m$, $m_3 = 200\,kg$

$E = 211\,GN/m^2 = 211 \times 10^9\,N/m^2$

To find:

Natural frequency of transverse vibrations (f_n)



Soln:-

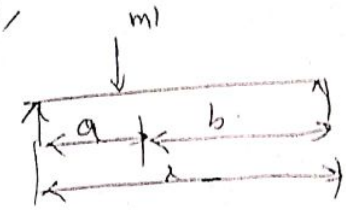
The shaft carrying the loads is shown in the fig:

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}}$$

For simply supported beam having point load,

deflection $\delta = \dfrac{W a^2 b^2}{3 E I l}$ | $I = \dfrac{\pi d^4}{64} = 5.153 \times 10^{-5}\ m^4$

Static deflection due to 250 kg mass,

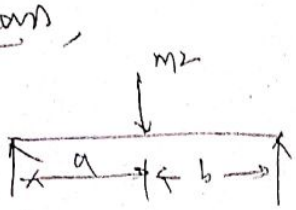


$$\delta_1 = \frac{(m_1 \times g)\, a^2 b^2}{3 E I l}$$

a = 0.6 m,
b = 1.9 m

$\Rightarrow \delta_1 = 3.9085 \times 10^{-5}$ m

Static deflection due to 500 kg mass,

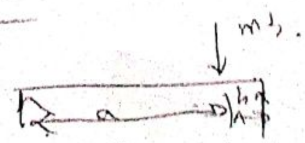


$$\delta_2 = \frac{(m_2 \times g)\, a^2 b^2}{3 E I l}$$

a = 1.5 m
b = 1 m

$\Rightarrow \delta_2 = 1.353 \times 10^{-4}$ m

Static deflection due to 200 kg mass



$$\delta_3 = \frac{(m_3 \times g)\, a^2 b^2}{3 E I l}$$

a = 2 m, b =

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$$\delta_3 = 2.4054 \times 10^{-5} \ m$$

Now, frequency of transverse vibrations by Dunkerley's formula,

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \underline{\underline{35.39 \ Hz}}$$

Whirling or Critical Speed of a shaft: (Whipping Speed)

Critical speed or whirling speed of a shaft is defined as the speed of a rotating shaft at which the shaft tends to vibrate violently in the transverse direction. This phenomenon is called whirling of shaft.

Cause:- Main cause is unbalanced mass.

Note:-

The critical speed or whirling speed of the shaft in r.p.s is equal to the natural frequency of transverse vibration in Hz.

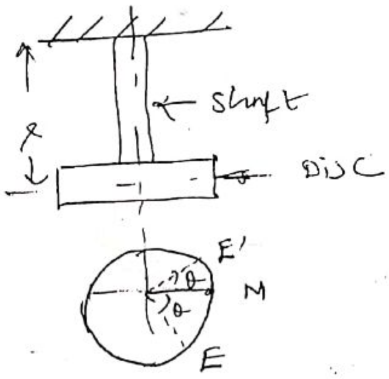
$$f_n \text{ in Hz} = N_c \text{ in rps.}$$

Torsional Vibrations

When the particles of a shaft or disc move in a circle about the axis of a shaft, then the vibrations are known as torsional vibrations. In this case, the shaft is twisted and untwisted alternately and torsional shear stresses are induced in the shaft.

Natural frequency of free torsional vibrations:

Consider a shaft of negligible mass whose one end is fixed and the other end carrying a disc as shown in fig.



M - Mean Position
E & E' - Extreme positions

Natural frequency,

$$\boxed{f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}}$$

where,

$\theta \Rightarrow$ Angular displacement of the shaft from mean position after time 't' in radians

$I \rightarrow$ mass moment of Inertia of disc in kg-m^2. ($I = m k^2$)

$m \rightarrow$ mass of disc in kg

$k \rightarrow$ Radius of gyration in 'm'

$q \rightarrow$ Torsional stiffness of the shaft in 'N-m'

Note:-

Torsional stiffness (q) is defined as the torque required to produce unit angular deflection.

Mathematically,

$$q = \frac{T}{\theta} = \frac{CJ}{l} \qquad \left[\because \frac{T}{J} = \frac{C\theta}{l} \leftarrow \text{torsional equation} \right]$$

$$\Rightarrow q = \frac{CJ}{l}.$$

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{Il}}}$$

$C \rightarrow$ modulus of rigidity

$J \rightarrow$ Polar moment of Inertia $= \frac{\pi d^4}{32}$

Inertia Effect of Mass of Shaft on Torsional Vibrations:-

When Inertia effect of mass of constraint (ie shaft) is considered then, the frequency of torsional vibrations is reduced.

ie, $$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I + \left(\frac{I_c}{3}\right)}}$$

$I_c = m_c k^2 \rightarrow$ Mass M.I of the constraint in kg-m²

$m_c \rightarrow$ mass of the constraint in 'kg'

$k_c \rightarrow$ Radius of gyration of the constraint in m

Torsionally Equivalent shaft

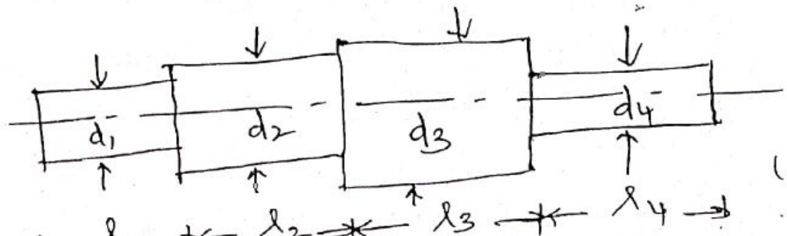
In many applications, the shaft of various diameters at different sections are used, as shown in fig. This shaft of varying diameters can be replaced by an equivalent shaft of uniform diameter. Because, finding the frequency of shaft of uniform diameter is more convenient than that of the shafts of various diameters.

Definition:-

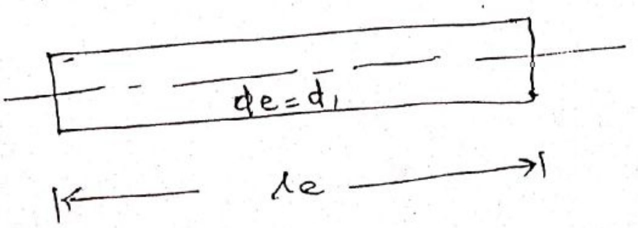
A torsionally equivalent shaft is one which has the same torsional stiffness as that of the stepped shaft so that it twists to the same extent under a given torque as that of the stepped shaft.

Derivations:-

Consider a shaft of varying diameters, as shown in fig. Let this shaft is replaced by an equivalent shaft of uniform diameter de and length le, as shown in fig.



(a) Shaft of varying diameters



(b) Torsionally equivalent shaft

The diameter 'de' is usually choosen as any one of the existing diameters of the actual shaft (i.e., stepped shaft). Let us assume that $de = d_1$, as shown in fig, then length of the equivalent shaft is given by,

$$\boxed{l_e = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4 + l_4\left(\frac{d_1}{d_4}\right)^4}$$

Note:-

If a stepped shaft (i.e., shaft of varying diameter) is given, it should be converted into a torsionally equivalent shaft, before solving any problems in torsional vibration systems.

Problems:-

1) A shaft of 100 mm diameter and 1 metre long has one of its end fixed and the other end carries a disc of mass 500 kg at a radius of gyration of 450 mm. The modulus of rigidity for the shaft material is $80\ GN/m^2$. Determine the frequency of torsional vibrations.

Given:-

$d = 100mm = 0.1m$; $l = 1m$, $m = 500kg$, $k = 450mm$
$G = 80 GN/m^2 = 80 \times 10^9\ N/m^2$. $= 0.45\ m$

To Find:- Frequency of torsional vibrations.

Soln:-

Frequency of torsional vibrations,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$$

torsional stiffness, $q = \frac{C \cdot J}{l}$

Polar moment of Inertia, $J = \frac{\pi d^4}{32} = 9.82 \times 10^{-6}\ m^4$

mass moment of Inertia, $I = m \cdot k^2 = 101.25\ kg\text{-}m^2$

$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{785.6 \times 10^3}{101.25}}$$

$$\Rightarrow \boxed{f_n = 14\ Hz}$$

2) A flywheel is mounted on a vertical shaft as shown in fig. The both ends of a shaft are fixed and its diameter is 50 mm. The flywheel has a mass of 500 kg and its radius of gyration is 0.5m. Find the natural frequency of torsional vibrations, if the modulus of rigidity for the shaft material is $80\ GN/m^2$.

Given:-

$d = 50mm = 50 \times 10^{-3}\ m$; $m = 500\ kg$; $k = 0.5m$

$q = 80\ GN/m^2 = 80 \times 10^9\ N/m^2$

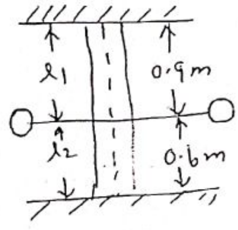


Fig.

To find:-

natural frequency of torsional vibrations

Soln :-

Natural frequency of torsional vibrations,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$$

Polar moment of Inertia of the shaft,

$$I = \frac{\pi d^4}{32} = 0.1 \times 10^{-6}\ m^4$$

Torsional stiffness of the shaft for length l_1,

$$q_1 = \frac{C\,J}{l_1} = 56 \times 10^3\ N-m$$

Torsional stiffness of the shaft for length l_2,

$$q_2 = \frac{C.J}{l_2} = 84 \times 10^3\ N-m$$

Total torsional stiffness of the shaft,

$$q = q_1 + q_2 = 140 \times 10^3\ N-m$$

Mass moment of Inertia,

$$I = m \cdot k^2 = 125\ kg - m^2$$

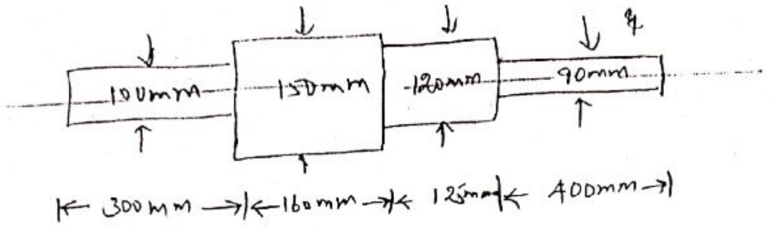
∴ Natural frequency of torsional vibrations,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{140 \times 10^3}{125}}$$

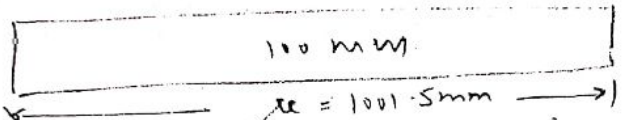
$$\Rightarrow \boxed{f_n = 5.32\ HZ}$$

AT

Reduce the stepped shaft shown in fig to a torsionally equivalent shaft of 100 mm diameter.



Given:-



$d_1 = 100$ mm ; $l_1 = 300$ mm ; $d_2 = 150$ mm ; $l_2 = 160$ mm ;
$d_3 = 120$ mm ; $l_3 = 125$ mm ; $d_4 = 90$ mm ; $l_4 = 400$ mm .

Soln:-

W.K.T, the length of the equivalent shaft,

$$l_e = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4 + l_4\left(\frac{d_1}{d_4}\right)^4$$

$$\Rightarrow l_e = 300 + 160\left(\frac{100}{150}\right)^4 + 125\left(\frac{100}{120}\right)^4 + 400\left(\frac{100}{90}\right)^4$$

$= 1001.5$ mm (or) 1.0015 m.

Unit-IV

Forced vibrations

- When the body vibrates under the influence of external force, then the body is said to be under forced vibrations.

Examples:- Ringing of electrical bell, air compressors, internal combustion engines, machine tools.

Types of External Excitation

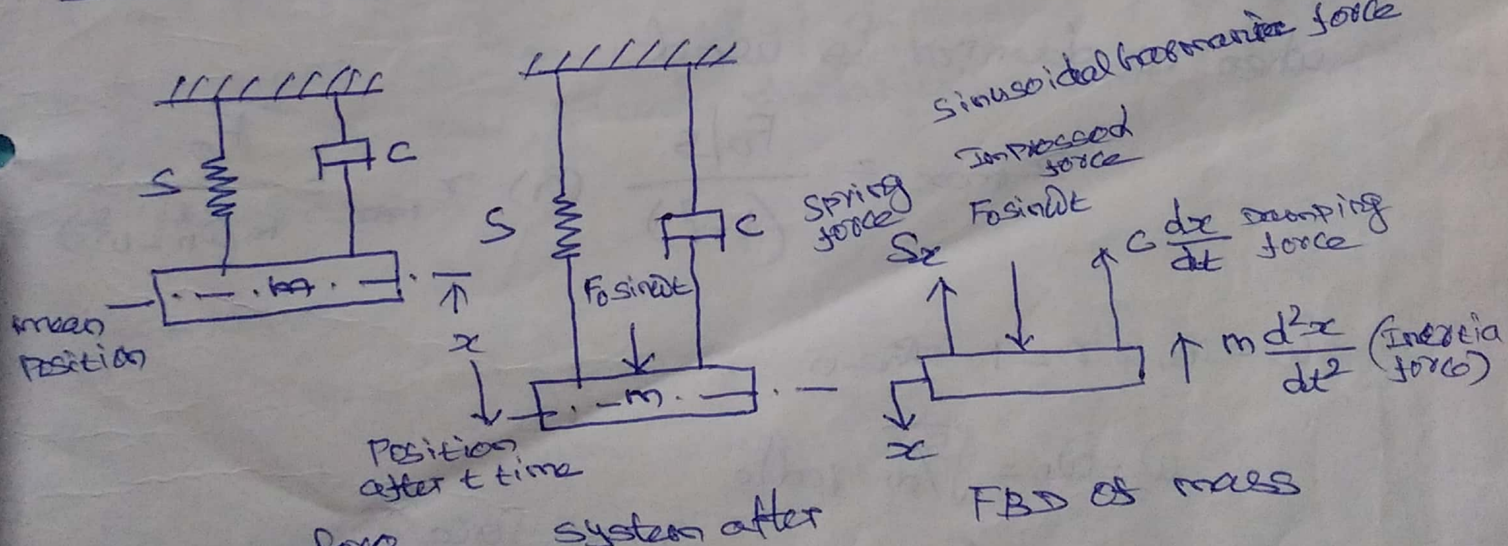
Three types of external forces applied are

i). periodic forces < harmonic / non-harmonic

ii). impulsive type of forces - transient

iii). Random forces - Earthquake & acoustic excitations

Forced vibrations with constant harmonic Excitation



System before applying force

system after applying force

FBD of mass

Characteristic equation of damped forced vibrations

To obtain complete solution

$$x = X e^{-\xi \omega_n t} \sin(\omega_d t + \phi_1) + \frac{F_0}{(s-m\omega^2)^2 + (c\omega)^2} \sin(\omega t - \phi)$$

Amplitude or maximum displacement of forced vibration

$$\cancel{x_{max} = \frac{F_0}{(s-m\omega^2)^2 + (c\omega)^2}}$$

$$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2) + (2\xi r)^2}}$$

r = Frequency ratio = ω/ω_n

Phase lag $\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$

when no damper is used

$$x_{max} = \frac{F_0/s}{(1-r^2)} \quad (\text{or}) \quad x_{max} = \frac{F_0}{m(\omega_n^2 - \omega^2)}$$

$\omega = \omega_n$, $r = 1$ then

$\omega = \omega_n = \sqrt{s/m}$ rad/s

$$x_{max} = \frac{F_0}{2\xi s}, \quad F_0 = s \cdot x_0$$

Static force

s - spring stiffness

x_0 - static deflection

magnification factor or Dynamic magnifier

The ratio of the maximum displacement of the forced vibration (x_{max}) to the static deflection due to static force [zero frequency deflection (x_0)] is known as magnification factor. It is denoted by M.F

$$x_{max} = M.F \times x_0$$

$$M.F = \frac{x_{max}}{x_0} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

①. A mass of 10 kg is suspended from one end of a helical spring, the other end being fixed. The stiffness of the spring is 10 N/mm. The viscous damping causes the amplitude to decrease to one-tenth of the initial value in four complete oscillations. If a periodic force of 150 cos 50 t N is applied at the mass in the vertical direction, find the amplitude of the forced vibrations? what is its value of resonance.

Given data :-

$m = 10\,kg$

$S = 10\,N/mm = 10 \times 10^3\,N/m$

$X_4 = \frac{1}{10} X_0$

$F = 150 \cos 50t$

$n = 4$

To find :-

1). Amplitude of the forced vibration

2). Amplitude of forced vibration at resonance

Solution :-

WKT, Amplitude of forced vibrations

$$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

To find ζ

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}, \quad \delta = 1/n \ln\left[\frac{X_0}{X_n}\right]$$

$$r = \omega/\omega_n, \quad \omega_n = \sqrt{S/m}$$

Amplitude of forced vibrations at resonance

$$x_{max} = \frac{F_0}{2\zeta . S}$$

$$F_0 \cos\omega t = 150 \cos 50 t$$

From the above equation

$$F_0 = 150N, \quad \omega = 50 \text{ rad/s}$$

$$r = \omega/\omega_n, \quad \omega_n = \sqrt{S/m} = \sqrt{\frac{10\times10^3}{10}} = 31.62 \text{ rad/s}$$

$$\therefore r = \frac{50}{31.62} = 1.58$$

$$\delta = \frac{1}{n} \ln\left(\frac{X_0}{X_n}\right) = \cancel{\frac{1}{4} \ln\left(\frac{X_0}{10 X_0}\right)}$$

$$X_4 = X_0 \cdot 1/10 = \frac{X_0}{10}$$

$$X_0/X_4 = 10$$

$$\delta = 1/n \ln\left(\frac{X_0}{X_n}\right) = 1/4 \ln(10) = 0.575$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$0.575 = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\sqrt{1-\zeta^2} = \frac{2\pi}{0.575}\zeta$$

$$\sqrt{1-\zeta^2} = 10.92\zeta$$

Take both side squaring

$$1-\zeta^2 = 119.24\zeta^2$$

$$1 = 119.24\zeta^2 + \zeta^2$$

$$1 = 120.24\zeta^2$$

$$\zeta = \sqrt{\frac{1}{120.24}} = 0.091$$

$$\therefore x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$= \frac{150/10\times10^3}{\sqrt{(1-1.58^2)^2 + (2\times0.091\times1.58)^2}}$$

$$= \frac{0.015}{2.239+0.082} = 6.46\times10^{-3}\,m$$

At resonance

$$x_{max} = \frac{F_0}{2\zeta s} = \frac{150}{2\times0.091\times10\times10^3} = 7.17\times10^{-3}\,m$$

Result:-

1). Amplitude of forced vibration $= 6.46\times10^{-3}\,m$

2). Amplitude of forced vibration at resonance $= 7.17\times10^{-3}\,m$

(5). A harmonic exciting force of 25N is acting on a machine part, which is having a mass of 2 kg and is vibrating in a viscous medium. This exciting force causes a resonant amplitude of 12.5mm with a period of 0.25 sec. Determine the damping coefficient. If the system is excited by a harmonic force of frequency 4 HZ, find the increase in amplitude of forced vibration when damper is removed.

Given data:-

$F_0 = 25\ N$

$m = 2\ kg$

Resonant $x_{max} = 12.5mm = 12.5 \times 10^{-3}$

$t_p = 0.25\ sec$

harmonic force of frequency $f = 4\ HZ$

TO find:-

1). Damping coefficient

2). Increase in amplitude of forced vibration when damper is removed.

Solution:-

WKT, damping coefficient $c = 2 m \omega_n \xi$

maximum amplitude of vibration at resonance

$$x_{max} = \frac{F_0}{2 \xi S}$$

$$\omega_n = \frac{2\pi}{t_p} \quad \text{or} \quad \omega_n = \sqrt{S/m} = \omega \text{ at resonance}$$

Increase in amplitude of vibration = Amplitude without damper − Amplitude with damper

Amplitude with damper $x_{max} = \dfrac{F_0/s}{\sqrt{(1-\gamma^2)^2 + (2\zeta\gamma)^2}}$

$\gamma = \omega/\omega_n$

$\omega_n = 2\pi f$

Amplitude without damper $x_{max} = \dfrac{F_0/s}{1-\gamma^2}$

$\omega_n = \dfrac{2\pi}{t_p} = \dfrac{2\pi}{0.2} = 31.416 \text{ rad/s}$

$\omega_n = \sqrt{s/m} \Rightarrow 31.416 = \sqrt{s/2}$

Taking both side squaring

$31.416^2 = \left(\sqrt{s/2}\right)^2$

$986.960 = s/2$

$s = 1973.92 \text{ N/m}$

$x_{max} = \dfrac{F_0}{2\zeta s} \Rightarrow 12.5\times10^{-3} = \dfrac{25}{2\times\zeta\times1973.92}$

$\zeta = \dfrac{25}{2\times12.5\times10^{-3}\times1973.92}$

$\zeta = 0.507$

$c = 2m\omega_n\zeta$

$= 2\times2\times31.416\times0.507$

$\boxed{c = 63.662 \text{ N/m/s}}$

Amplitude with damper

$x_{max} = \dfrac{F_0/s}{\sqrt{(1-\gamma^2)^2 + (2\zeta\gamma)^2}}$

$\gamma = \omega/\omega_n$, $\omega_n = 2\pi f = 2\times\pi\times4 = 25.133 \text{ rad/s}$

$$\gamma = \frac{\omega}{\omega_n} = \frac{25.133}{31.416} = 0.8$$

$$\therefore x_{max} = \frac{(25/1973.92)}{\sqrt{(1-(0.8)^2)^2 + (2\times 0.507\times 0.8)^2}}$$

$$x_{max} = 0.014\,m$$

Amplitude without damper

$$x_{max} = \frac{F_0/s}{1-\gamma^2} = \frac{(25/1973.92)}{1-(0.8)^2}$$

$$x_{max} = 0.035\,m$$

Increase in amplitude $= 0.035 - 0.014$

$= 0.021\,m$

Result:-

1). Damping coefficient (C) = 63.662 N/m/s

2). Increase in amplitude = 0.021 m

Forcing caused by unbalance

(Forced vibrations with Rotating and Reciprocating unbalance)

Almost in all rotating and reciprocating machinery ~~bikes~~ (electric motor, turbine & I.C engine) have some amount of unbalanced force left in them even after correcting their unbalance on precision balancing machines. This small amount of unbalanced force produces the exciting force in a machine.

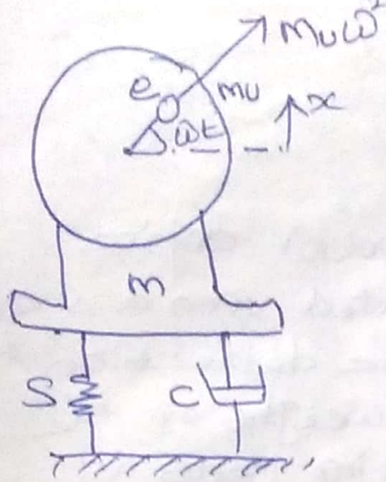
Let, m - vibrating mass

m_u - unbalanced mass

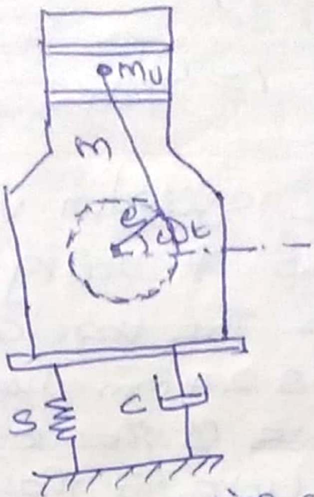
e - eccentricity

s - spring stiffness

c - Damping coefficient



Rotating unbalance



Reciprocating unbalance

Governing equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + cx = m_u e\omega^2 \sin\omega t$$

Complete solution

$$x = x_c + x_p$$

$$x = Xe^{-\zeta\omega_n t}\sin(\omega_d t + \phi_1) + \frac{m_u\omega^2 e}{\sqrt{(S-m\omega^2)^2 + (c\omega)^2}} \cdot \cos(\omega t - \phi$$

The amplitude of forced vibration caused by unbalance

$$x_{max} = \frac{m_u\omega^2 e}{\sqrt{(S-m\omega^2)^2 + (c\omega)^2}}$$

(or)

$$\frac{x_{max}}{\left(\frac{m_u e}{m}\right)} = \frac{(\omega/\omega_n)^2}{\sqrt{\left(1-(\omega/\omega_n)^2\right)^2 + \left[2\zeta(\omega/\omega_n)\right]^2}}$$

(or)

$$\frac{x_{max}}{\left(\frac{m_u . e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

r = Frequency ratio = ω/ω_n

Phase angle $\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$

$r = \omega/\omega_n$

① A single cylinder vertical petrol engine of total mass of 200 kg is mounted upon a steel chassis frame. The vertical static deflection of the frame is 2.4 mm due to the weight of the engine. The mass of the reciprocating parts is 18 kg and the stroke of the piston is 160 mm with SHM. If dashpot of damping coefficient 1 N/mm/s is used to dampen the vibrations, calculate at steady state: i). the amplitude of forced vibrations at 500 rpm engine speed and ii). the speed of the driving shaft at which resonance will occur.

Given data:-

$m = 200\,kg$

Static deflection $\delta = 2.4\,mm = 2.4\times10^{-3}\,m$

$m_0 = 18\,kg$

Stroke $L = 160\,mm = 0.16\,m$

$C = 1\,N/mm/s = 1\times10^{3}\,N/m/s$

$N = 500\,rpm$

To find:-

1). Amplitude of steady state forced vibrations

2). Speed of the driving shaft at which resonance will occur.

Solution:-

Amplitude of steady state forced vibrations

$$\frac{x_{max}}{\left(\frac{m_0 \cdot e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$r = \omega/\omega_n$

$\omega = 2\pi N/60 = (2\pi \times 500)/60$

$\omega = 52.36$ rad/s

$\omega_n = \sqrt{s/m} = \sqrt{g/\delta} = \sqrt{\dfrac{9.81}{2.4\times10^{-3}}}$ $\therefore r = \omega/\omega_n = \dfrac{52.36}{63.934}$

$r = 0.819$

$\omega_n = 63.934$ rad/s

eccentricity $e = \dfrac{\text{stroke}}{2} = \dfrac{0.160}{2} = 0.080\,m$

$\zeta = \dfrac{c}{2m\omega_n} = \dfrac{1\times10^{3}}{2\times200\times63.934}$

$\zeta = 0.039$

$$\therefore \frac{x_{max}}{\left(\frac{m_u \cdot e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$\frac{x_{max}}{\left(\frac{18\times0.080}{200}\right)} = \frac{(0.819)^2}{\sqrt{(1-0.819^2)^2+(2\times0.039\times0.819)^2}}$$

$$\frac{x_{max}}{0.007} = \frac{0.671}{0.335} \Rightarrow \frac{x_{max}}{0.007} = 2.003$$

$x_{max} = 0.007\times2.003 = 0.014\,m$

<u>speed of the driving shaft at which resonance will occur</u>

At resonance

$\omega = \omega_n$

$\omega = \dfrac{2\pi N}{60} \Rightarrow N = \dfrac{60\times\omega}{2\pi} = \dfrac{60\times63.934}{2\pi}$

$N = 610.52$ rpm say 611 rpm

<u>Result:-</u>

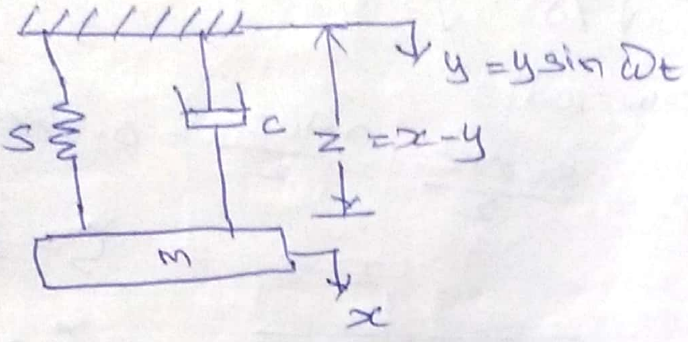
1). Amplitude of steady state forced vibrations = 0.014 m

2). speed of the driving shaft at which resonance will occur = 611 rpm

<u>4/9/19</u>

1, 3, 7, 10, 12, 22, 24, 23, 25, 27, 30, 31, 32, 40, 41, 42, 45, 48, 49.

Forced vibrations due to excitation of the support (Support motion)



Absolute amplitude

Equation of motion

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + Sx = y\sqrt{S^2 + (c\omega)^2}\cdot\sin(\omega t + \alpha)$$

The amplitude of vibration

$$\frac{x_{max}}{y} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

where, $r = \omega/\omega_n$

Phase angle $\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$

Relative amplitude

Equation of motion

$$m\frac{d^2z}{dt^2} + c\frac{dz}{dt} + Sz = m\omega^2 Y\sin\omega t$$

Relative amplitude of vibration

$$\frac{z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

Phase angle $\phi = \tan^{-1}\left(\frac{2\xi\gamma}{1-\gamma^2}\right)$

Dynamic load on isolators due to vibrations $F_{dyn} = Z\sqrt{S^2+(C\omega)^2}$

—x—

①. A computer monitor set of 18 kg mass must be isolated from a machine vibrating with an amplitude 0.06 mm at 520 c.p.m. The set is mounted on four isolators, each having a spring scale of 31000 N/m and damping coefficient of 400 N-sec/m. a). what is the amplitude of vibration of the computer monitor? b). what is the dynamic load on each isolator due to vibration.

Given data:-

$m = 18\ kg$

$Y = 0.06\,mm = 0.06 \times 10^{-3}\ m$

$N = 520\ CPm$

Number of isolators $= 4$

$S = 31000\ N/m$

$C = 400$ ~~N-sec~~ $N.s/m$

To find:

1). Amplitude of the computer monitor

2). Dynamic load on each isolator due to vibration

Solution:

WKT, Amplitude of vibration

$$\frac{x_{max}}{Y} = \frac{\sqrt{1+(2\xi\gamma)^2}}{\sqrt{(1-\gamma^2)^2+(2\xi\gamma)^2}}$$

Dynamic load on ~~each~~ isolators $F_{dyn} = Z\sqrt{S^2+(C\omega)^2}$

$$\frac{Z}{Y} = \frac{\gamma^2}{\sqrt{(1-\gamma^2)^2+(2\xi\gamma)^2}}$$

since there are 4 isolators, then equivalent stiffness and damping coefficient is given by

$$S_{eq} = 4 \times S = 4 \times 31000 = 124000 \text{ N/m}$$

$$C_{eq} = 4 \times C = 4 \times 400 = 1600 \text{ N/m/s}$$

$$\gamma = \frac{\omega}{\omega_n}, \quad \xi = \frac{c}{2m\omega_n}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 520}{60} = 54.45 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{124000}{18}} = 82.999 \text{ rad/sec}$$

$$\gamma = \omega/\omega_n = \frac{54.45}{8.999} = 0.656$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{1600}{2 \times 18 \times 82.999} = 0.535$$

$$\therefore \frac{x_{max}}{y} = \frac{\sqrt{1 + (2\zeta\gamma)^2}}{\sqrt{(1-\gamma^2)^2 + (2\zeta\gamma)^2}}$$

$$\frac{x_{max}}{0.06 \times 10^{-3}} = \frac{\sqrt{1 + (2 \times 0.535 \times 0.656)^2}}{\sqrt{(1-0.656^2)^2 + (2 \times 0.535 \times 0.656)^2}} = \frac{1.222}{0.904}$$

$$x_{max} = \frac{1.222}{0.904} \times 0.06 \times 10^{-3}$$

$$x_{max} = 8.11 \times 10^{-5} \text{ m}$$

$$F_{Dyn} = Z\sqrt{S^2 + (C\omega)^2}$$

$$\frac{Z}{Y} = \frac{\gamma^2}{\sqrt{(1-\gamma^2)^2 + (2\zeta\gamma)^2}} = \frac{0.656^2}{0.904}$$

$$\frac{Z}{Y} = 0.476$$

$z = 0.476 \times y = 0.476 \times 0.06 \times 10^{-3}$

$z = 2.85 \times 10^{-5}$ m

$\therefore F_{dyn} = z\sqrt{s^2 + (c\omega)^2}$

$= 2.85 \times 10^{-5}\sqrt{(124000)^2 + (1600 \times 54.45)^2}$

$F_{dyn} = 4.31$ N

The dynamic load on each isolator $= \frac{4.31}{4} = 1.079$ N

Result:-

1). Amplitude of computer monitor $= 8.11 \times 10^{-5}$ m

2). Dynamic load on each isolator $= 1.079$ N

—×—

Vibration isolation

The process of reducing the vibrations of machines and hence reducing the transmitted force to the foundation using vibration isolating materials is called vibration isolation.

Isolating materials

Rubber, felt, cork and metallic springs

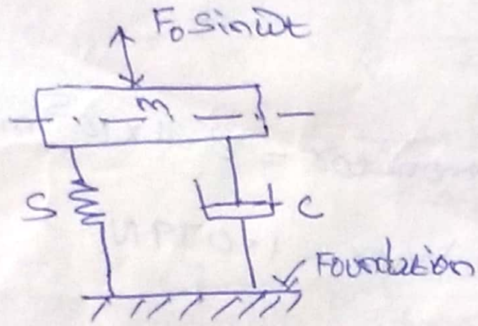
Transmissibility

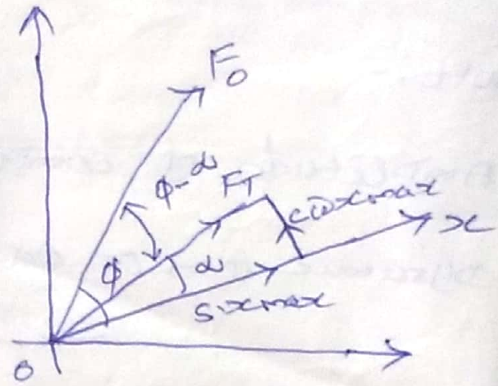
– measure of the effectiveness of the vibration isolating material.

Transmissibility ratio

- The ratio of the force transmitted (F_T) to the force applied (F_0) on the system. Also known as isolating factor. It is denoted by ε (epsilon)

$$\text{Transmissibility ratio} = \varepsilon = \frac{\text{Force transmitted to the foundation}}{\text{Force applied on the system}} = \frac{F_T}{F_0}$$





Consider a mass m supported on the foundation by means of an isolator.

Let the mass is excited by a simple harmonic force

$$F = F_0 \sin \omega t$$

The force transmitted to the foundation consists of the following two forces:

1). spring or elastic force $= S \cdot x_{max}$

2). Damping force $= C \cdot \omega \cdot x_{max}$

$$\therefore F_T = \sqrt{(S \cdot x_{max})^2 + (C \cdot \omega \cdot x_{max})^2}$$

$$= x_{max} \sqrt{S^2 + C^2 \omega^2}$$

$$\therefore \varepsilon = \frac{F_T}{F_0} = x_{max} \sqrt{S^2 + C^2 \omega^2} / F_0$$

$$\therefore \varepsilon = \frac{F_T}{F_0} = \frac{\sqrt{1+(2\xi\gamma)^2}}{\sqrt{(1-\gamma^2)^2+(2\xi\gamma)^2}}$$

1) At resonance $\omega = \omega_n$

$$\varepsilon = \frac{\sqrt{1+(2\xi)^2}}{2\xi}$$

2). $\xi = 0$

$$\varepsilon = \pm \frac{1}{(1-\gamma^2)}$$

Phase lag

$$\alpha = \tan^{-1}(2\xi\gamma) \quad , \quad \phi = \tan^{-1}\left(\frac{2\xi\gamma}{1-\gamma^2}\right)$$

$$\phi - \alpha = \tan^{-1}\left(\frac{2\xi\gamma}{1-\gamma^2}\right) - \tan^{-1}(2\xi\gamma)$$

motion or Amplitude transmissibility

$$\varepsilon_A = \frac{x_{max}}{Y} = \frac{\sqrt{1+(2\xi\gamma)^2}}{\sqrt{(1-\gamma^2)^2+(2\xi\gamma)^2}}$$

—x—

①. A machine 100 kg has a 20 kg rotor with 0.5 mm eccentricity. The mounting springs have $s = 85 \times 10^3$ N/m. The damping ratio is 0.02. The operating speed is 600 rpm and the unit is constrained to move vertically. Find i). dynamic amplitude of the machine ii). force transmitted to the supports.

Given data:-

$m = 100\,kg$, $m_0 = 20\,kg$ $e = 0.5\,mm = 0.5 \times 10^{-3}\,m$

$S = 85 \times 10^{3}\,N/m$, $\zeta = 0.02$ $N = 600\,rpm$

To find:-

1). Dynamic amplitude of machine

2). Force transmitted to the supports

Solution:

1). Dynamic amplitude of the machine

$$\text{WKT, } x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$r = \omega/\omega_n,\ \omega = \frac{2\pi N}{60},\ \omega_n = \sqrt{S/m}$$

$$F_0 = m_0 \cdot \omega^2 \cdot e$$

2). Force transmitted to the supports (F_T)

$$\text{WKT, } \varepsilon = \frac{F_T}{F_0} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$F_T = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \times F_0$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60} = 62.83\ rad/s$$

$$\omega_n = \sqrt{S/m} = \sqrt{\frac{85 \times 10^{3}}{100}} = 29.15\ rad/s$$

$$r = \omega/\omega_n = \frac{62.83}{29.15} = 2.155$$

$$F_0 = m_0 \omega^2 e = 20 \times (62.83)^2 \times 0.5 \times 10^{-3}$$

$F_0 = 39.48\,N$

$$\therefore x_{max} = \frac{F_0/s}{\sqrt{(1-\gamma^2)^2 + (2\zeta\gamma)^2}}$$

$$= \frac{39.48/85\times10^3}{\sqrt{(1-2.155^2)^2 + (2\times0.02\times2.155)^2}}$$

$$= \frac{4.644\times10^{-4}}{\sqrt{4.644 + 7.43\times10^{-3}}} = 1.274\times10^{-4}\,m$$

$$\therefore F_T = \frac{\sqrt{1+(2\zeta\gamma)^2}}{(1-\gamma^2)^2+(2\zeta\gamma)^2} \times F_0$$

$$= \frac{\sqrt{1+(2\times0.02\times2.155)^2}}{\sqrt{(1-2.155^2)^2+(2\times0.02\times2.155)^2}} \times 39.48$$

$$= 10.87\,N$$

Result:-

1). Dynamic amplitude of machine $= 1.274\times10^{-4}\,m$

2). Force transmitted to the support $= 10.87\,N$

(2). A machine of mass 75 kg is mounted on springs of stiffness 12×10^5 N/m and with an assumed damping factor of 0.2. A piston within the machine of mass 2 kg has a reciprocating motion with a stroke of 80 mm and a speed of 3000 cycles/min. Assuming the motion to be simple harmonic, find; i). the amplitude of motion of the machine, ii). its phase angle with respect to the exciting force

iii). the force transmitted to the foundation
iv). the phase angle of transmitted force with respect to exciting force v). the phase lag of the transmitted force with respect to the applied force.

Given data:-

$m = 75\ kg$

$S = 12 \times 10^5\ N/m$

$\zeta = 0.2$

$m_u = 2kg$

$Stroke = 80mm = 0.08m$

$N = 3000 rpm$

To find:-

1). The amplitude of motion of machine (x_{max})

2). Phase angle with respect to the exciting force (ϕ)

3). The force transmitted to the foundation (F_T)

4). Phase angle of the transmitted force wrt. the exciting force (α)

5). Phase lag of the transmitted force with respect to the applied force ($\phi - \alpha$)

Solution:-

WKT. 1). $x_{max} = \dfrac{F_0/S}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

$F_0 = m_u \omega^2 e$, $r = \omega/\omega_n$, $\omega = 2\pi N/60$, $\omega_n = \sqrt{S/m}$

$e = \dfrac{Stroke}{2}$

2). $\phi = \tan^{-1}\left(\dfrac{2\zeta r}{1-r^2}\right)$

3). $F_T \Rightarrow \frac{F_T}{F_0} = \varepsilon$

$$F_T = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \times F_0$$

4). $\alpha = \tan^{-1}(2\zeta r)$

~~5). ϕ ...~~ $e = \frac{0.08}{2} = 0.04\,m$

$\omega = 2\pi N/60 = \frac{2\pi \times 3000}{60} = 314.16\,rad/sec$

$\omega_n = \sqrt{s/m} = \sqrt{\frac{12\times10^5}{75}} = 126.49\,rad/sec$

$r = \omega/\omega_n = \frac{314.1}{126.49} = ~~0.248~~\ 2.483$

$F_0 = m_u \cdot \omega^2 \cdot e = 2 \times 314.16^2 \times 0.04 = 7895.72\,N$

By using the above formula & values we get the result.

Result:-

1). $x_{max} = 1.25 \times 10^{-3}\,m$

2). $\phi = 169.12^\circ$

3). $\alpha = 44.81^\circ$

4). $F_T = 2114.62\,N$

5). $\phi - \alpha = 124.31^\circ$

③. An industrial machine weighing 445 kg is supported on a spring with a statical deflection of 0.5 cm. If the machine has rotating imbalance of 25 kg-cm, determine the force transmitted at 1200 rpm and the dynamic amplitude at that speed.

Given data:-

$$m = 445\,kg$$

$$\delta = 0.5\,cm = 0.5 \times 10^{-2}\,m$$

$$m_u \cdot e = 25\,kg \cdot cm = 25 \times 10^{-2}\,kg \cdot m$$

$$N = 1200\,rpm$$

To find:-

1). Force transmitted (F_T)

2). Dynamic amplitude at 1200 rpm (x_{max})

Solution:-

1). Force transmitted, when there is no damper $(\zeta = 0)$ WKT

$$\varepsilon = \frac{F_T}{F_0} = \pm \frac{1}{1-\gamma^2}$$

$$\gamma = \frac{\omega}{\omega_n}, \quad \omega = 2\pi N/60, \quad \omega_n = \sqrt{g/\delta}$$

$$F_0 = (m_u \cdot e) \cdot \omega^2$$

2). Dynamic amplitude at 1200 rpm

WKT, The amplitude of vibration when there is no damper,

$$x_{max} = \frac{F_0/s}{1-\gamma^2}$$

$$\therefore \omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1200}{60} = 125.66\,rad/s$$

$$\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{0.5 \times 10^{-2}}} = 44.29\,rad/sec$$

$$\gamma = \frac{\omega}{\omega_n} = \frac{125.66}{44.29} = 2.837$$

$\therefore \varepsilon = \frac{F_T}{F_0}$ $F_0 = (m.e) \cdot \omega^2$

$= (25 \times 10^{-2}) \times (125.66)^2$

$= 3947.6\,N$

$$\varepsilon = \frac{F_T}{F_0} = \pm \frac{1}{1-r^2}$$

when, $r > 1$, then $\frac{F_T}{F_0} = \frac{1}{r^2 - 1}$

$$\frac{F_T}{3947.6} = \frac{1}{2.837^2 - 1} = 0.142$$

$$F_T = 3947.6 \times 0.142 = 560.05\,N$$

$$x_{max} = \frac{F_0/S}{1-r^2}$$

$$\omega_n = \sqrt{\frac{S}{m}} \Rightarrow 44.29 = \sqrt{\frac{S}{445}}$$

$$44.29^2 = \left(\sqrt{\frac{S}{445}}\right)^2$$

$$1961.06 = \frac{S}{445} \Rightarrow S = 1961.06 \times 445$$

$$S = 872671.7\,N/m$$

$$\therefore x_{max} = \frac{3947.6/872671.7}{1 - 2.837^2} = \frac{0.0045}{7.048}$$

$$x_{max} = 6.384 \times 10^{-4}\,m$$

Result:-

1). Force transmitted $(F_T) = 560.05\,N$

2). Dynamic amplitude at 1200 rpm $(x_{max}) = 6.384 \times 10^{-4}\,m$

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$$20.53^2 = \left(\sqrt{\frac{S}{100}}\right)^2$$

$$421.48 = \frac{S}{100} \Rightarrow S = 421.48 \times 100$$

$$S = 42148.09 \ N/m$$

$$\gamma = \frac{\omega}{\omega_n} = \frac{104.71}{20.53} = 5.1$$

$$F_O = m_u \cdot e \ \omega^2$$

$$= 2 \times \left(\frac{0.08}{2}\right) \times 104.71^2$$

$$= 877.13 \ N$$

$$\therefore F_T = \varepsilon \cdot F_O = 0.04 \times 877.13$$

$$\boxed{F_T = 35.08 N}$$

At resonance

$$\varepsilon = \frac{\sqrt{1+(2\xi)^2}}{2\xi}$$

$$\ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\ln\left(\frac{x_0}{0.75 x_0}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$0.287 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\sqrt{1-\xi^2} = \frac{2\pi\xi}{0.287} = 21.89\xi$$

Taking square on both side

$$1-\xi^2 = (21.89\xi)^2$$

$$1-\xi^2 = 479.28\xi^2$$

$$1 = 480.28\xi^2$$

$$\xi^2 = 0.0028$$

Taking ~~root~~ on both side

$\zeta = \sqrt{0.0028}$

$\zeta = 0.0456$

$$\therefore \varepsilon = \frac{\sqrt{1+(2\zeta)^2}}{2\zeta} = \frac{\sqrt{1+(2\times0.0456)^2}}{2\times0.0456}$$

$\varepsilon = 11.01$

$\therefore F_T = \varepsilon . F_0 = \cancel{11.01} \times \cancel{\ldots}$

At resonance

$$F_0 = m_u . e (\omega_n)^2 = 2\times\left(\frac{0.08}{2}\right)\times(20.53)^2$$

$F_0 = 33.718\,N$

$\therefore F_T = 11.01 \times 33.718$

$$\boxed{F_T = 371.24\,N}$$

$$\left.\begin{array}{r}\text{Amplitude of the forced vibration}\\ \text{at resonance}\end{array}\right\} = \frac{\text{Force transmitted at resonance}}{\text{Combine stiffness}}$$

$$= \frac{371.24}{42148.09}$$

$$= 0.0088\,m$$

Result:-

1). Force transmitted $= 35.08\,N$

2). Force transmitted at resonance $= 371.24\,N$

3). Amplitude of the vibrations at resonance $= 0.0088\,m$